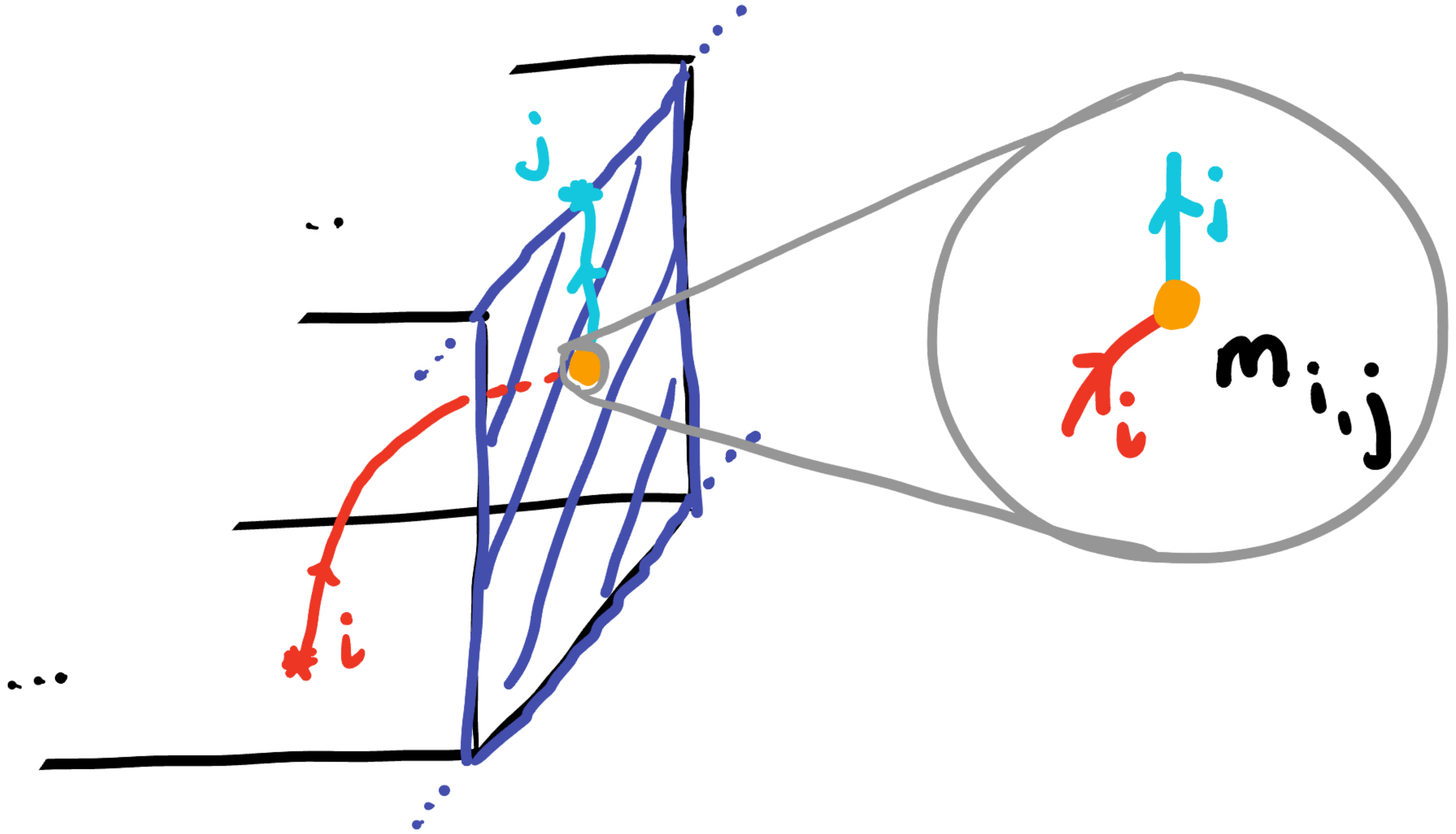
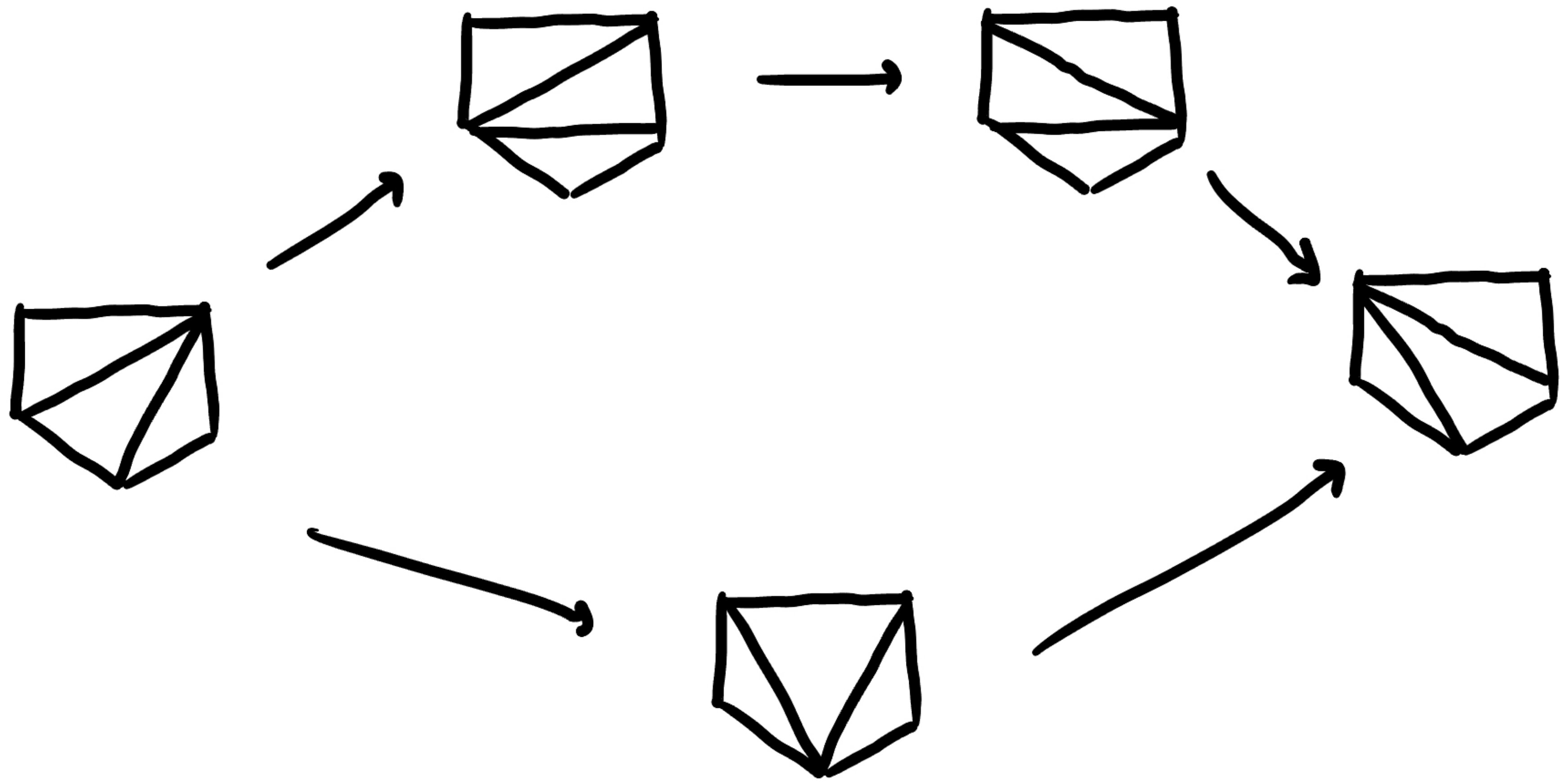
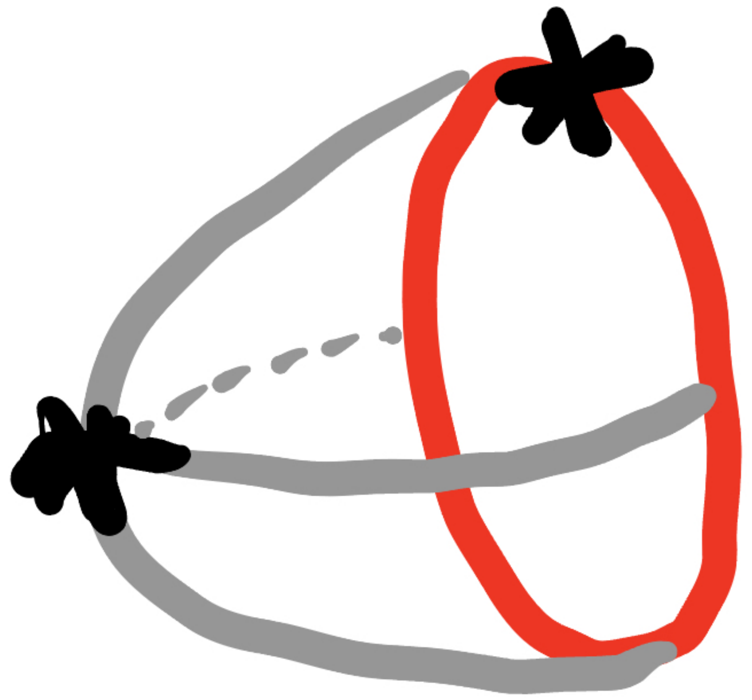






bulk-to-boundary anyon fusion

from microscopic models

arXiv:2302.01835

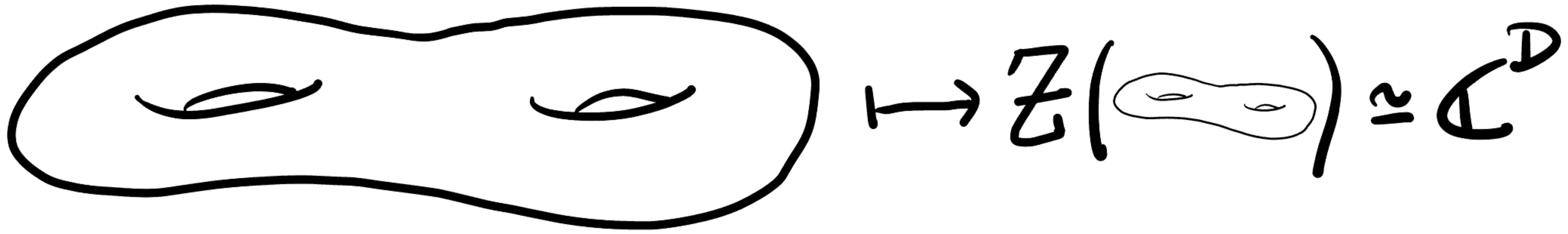
joint work with
Andreas Bauer and Jens Eisert

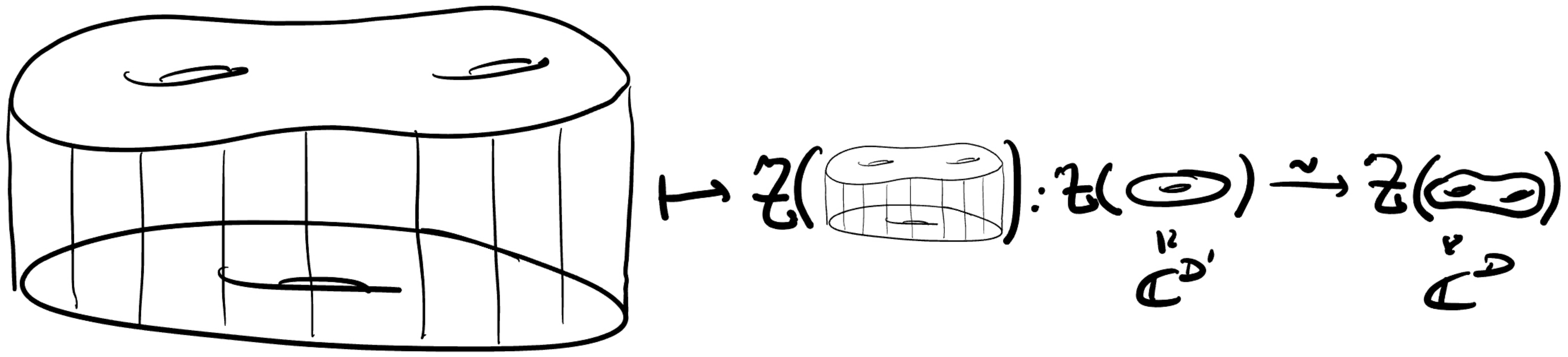
Julio C. Magdalena de la Fuente, TOPO 2023, Tübingen

Freie Universität Berlin

2+1d topological theories with defects

macroscopic theory





“2-manifolds to state(space)”

“3-bordisms to linear maps”

Atiyah 1988

2+1d topological theories with defects

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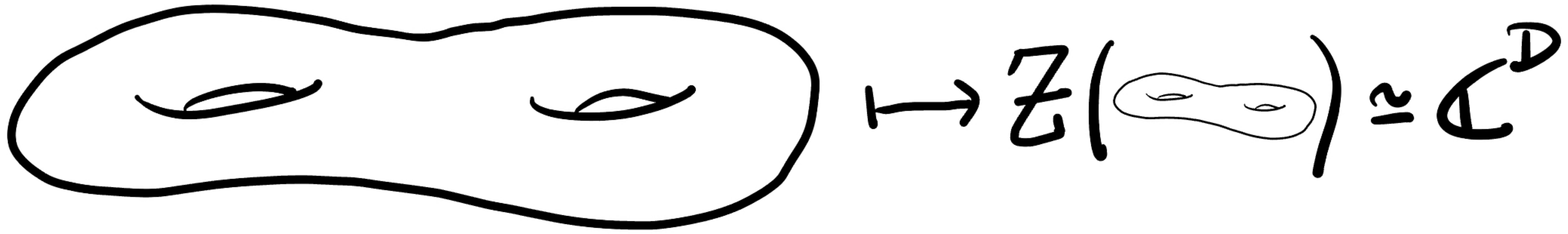
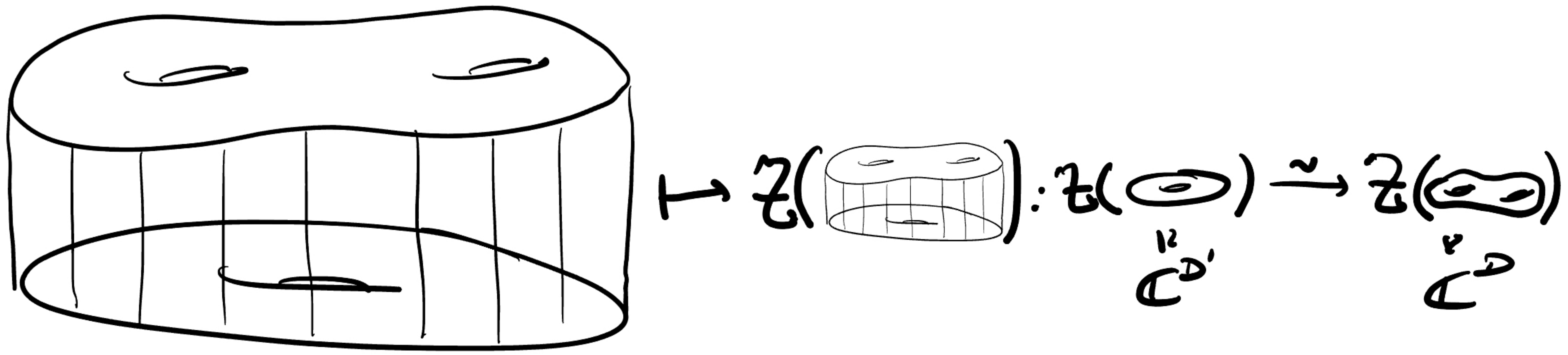


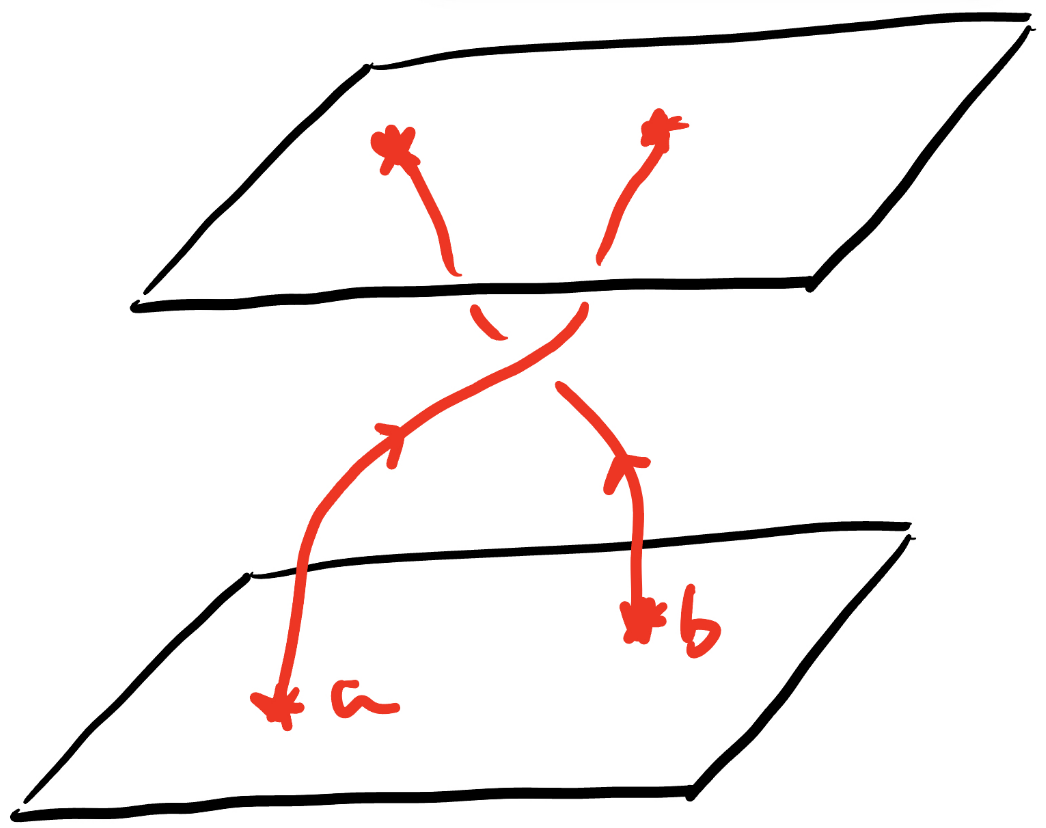
"2-manifolds to state(space)"



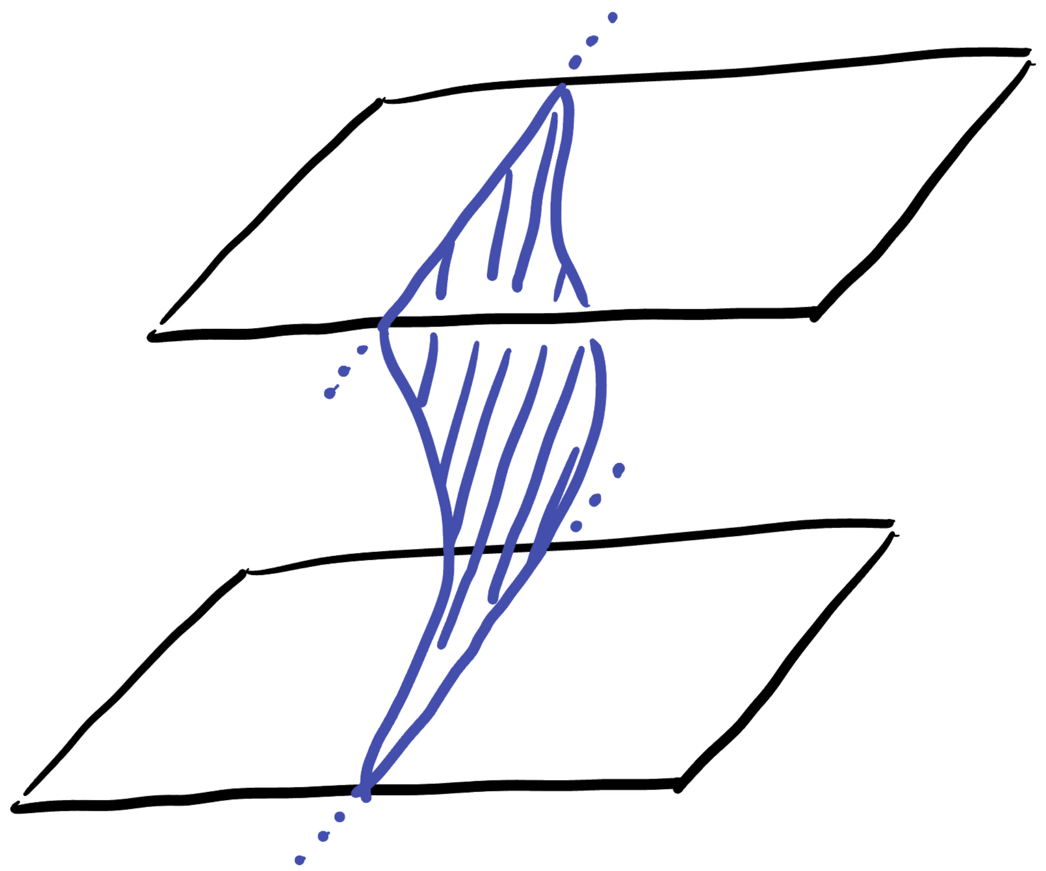
"3-bordisms to linear maps"

Atiyah 1988

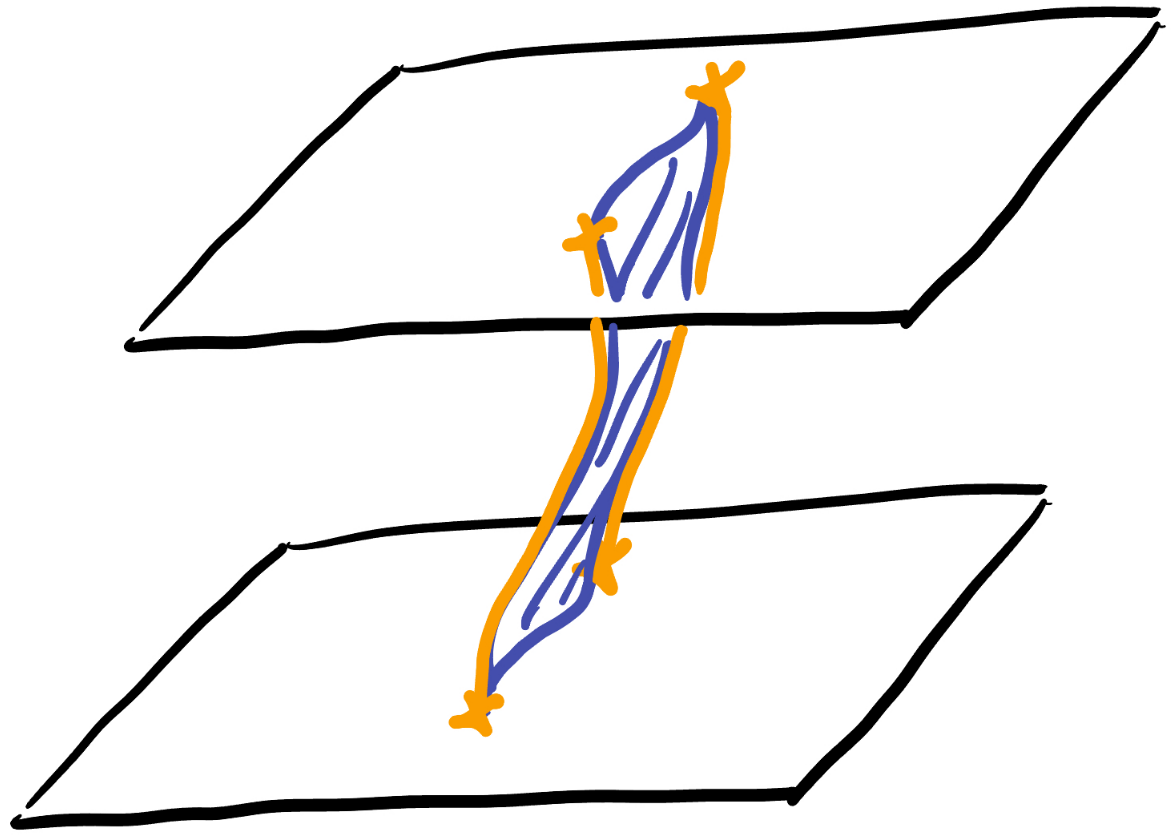
can be *extended* with defects



anyons



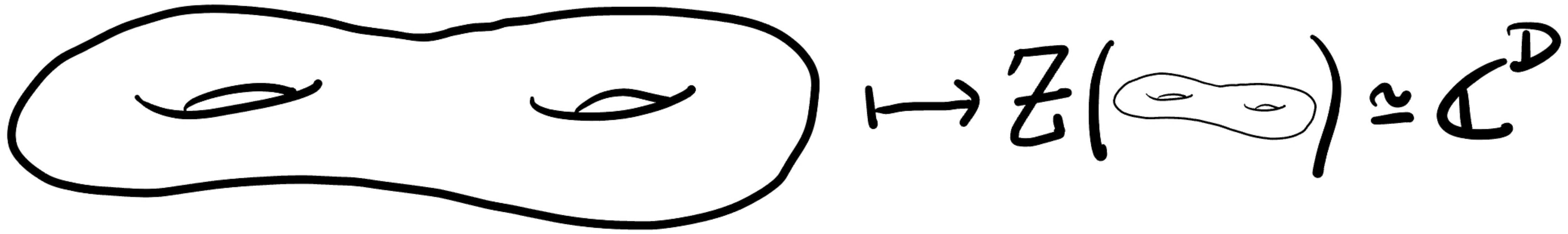
domain walls



twist defects

2+1d topological theories with defects

macroscopic theory



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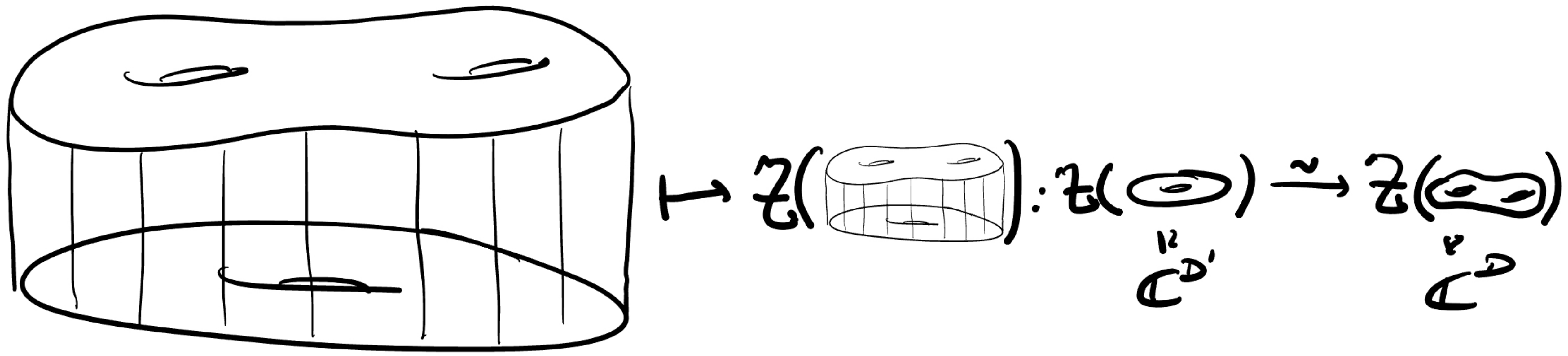

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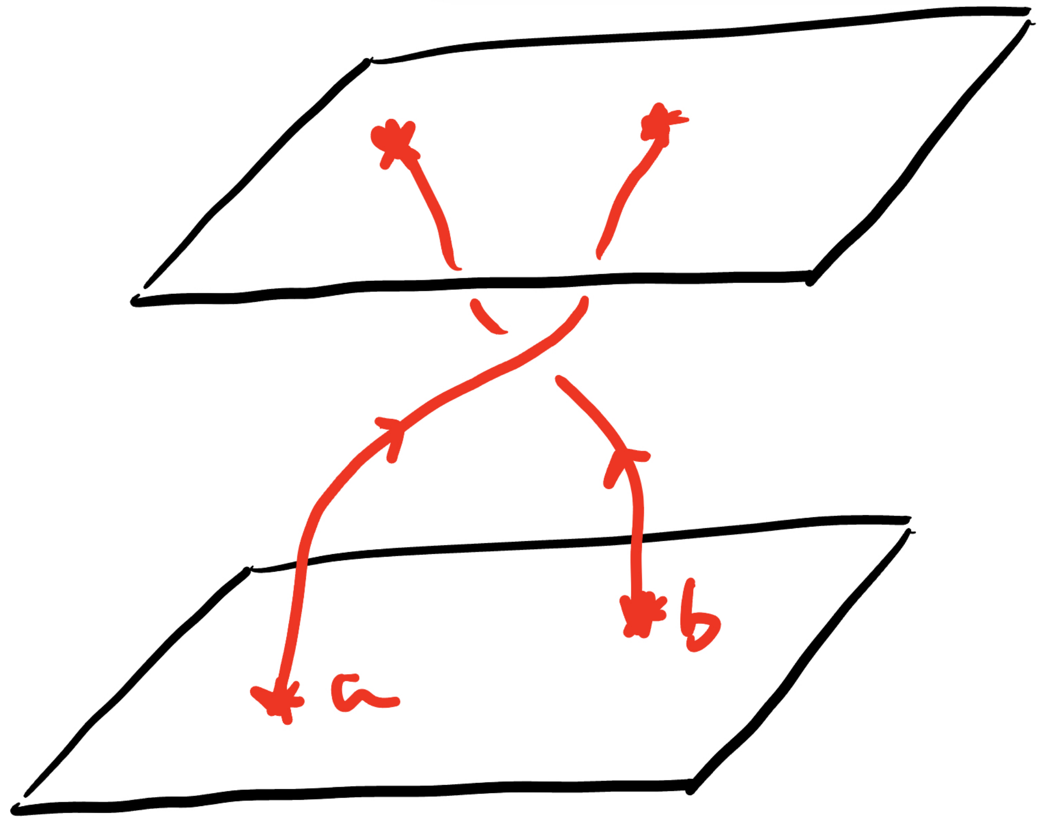

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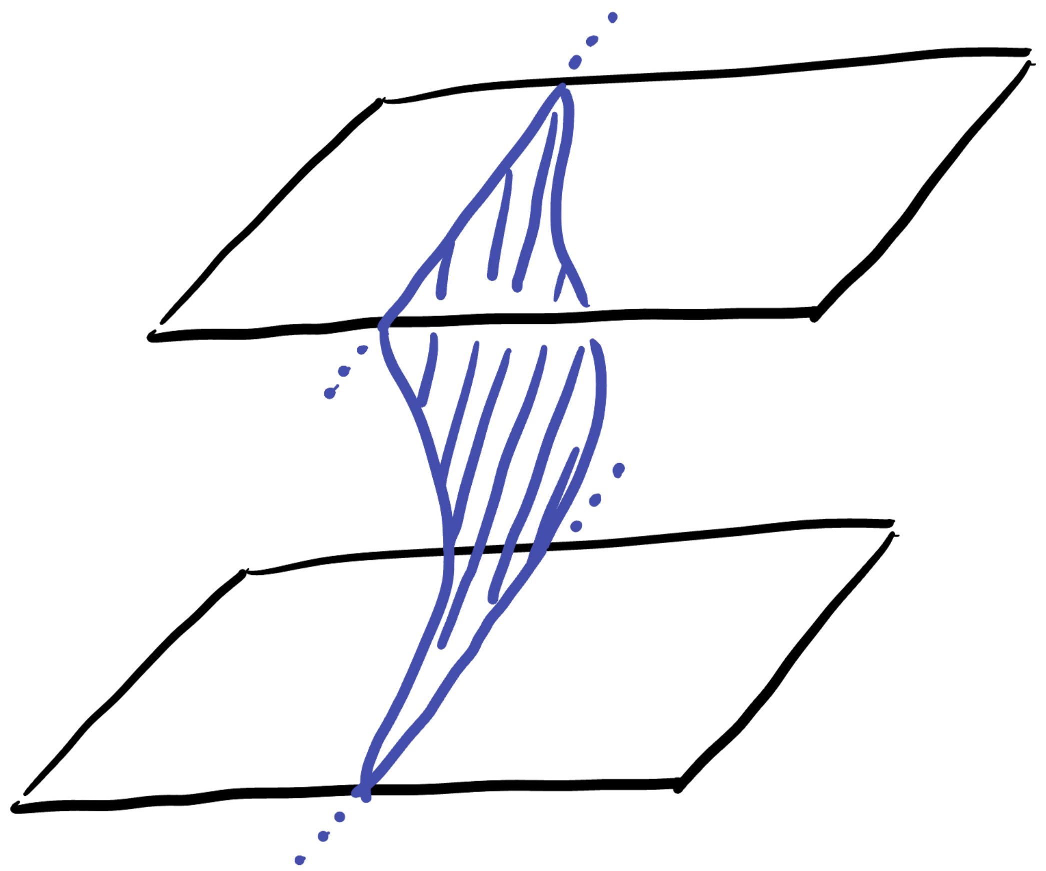

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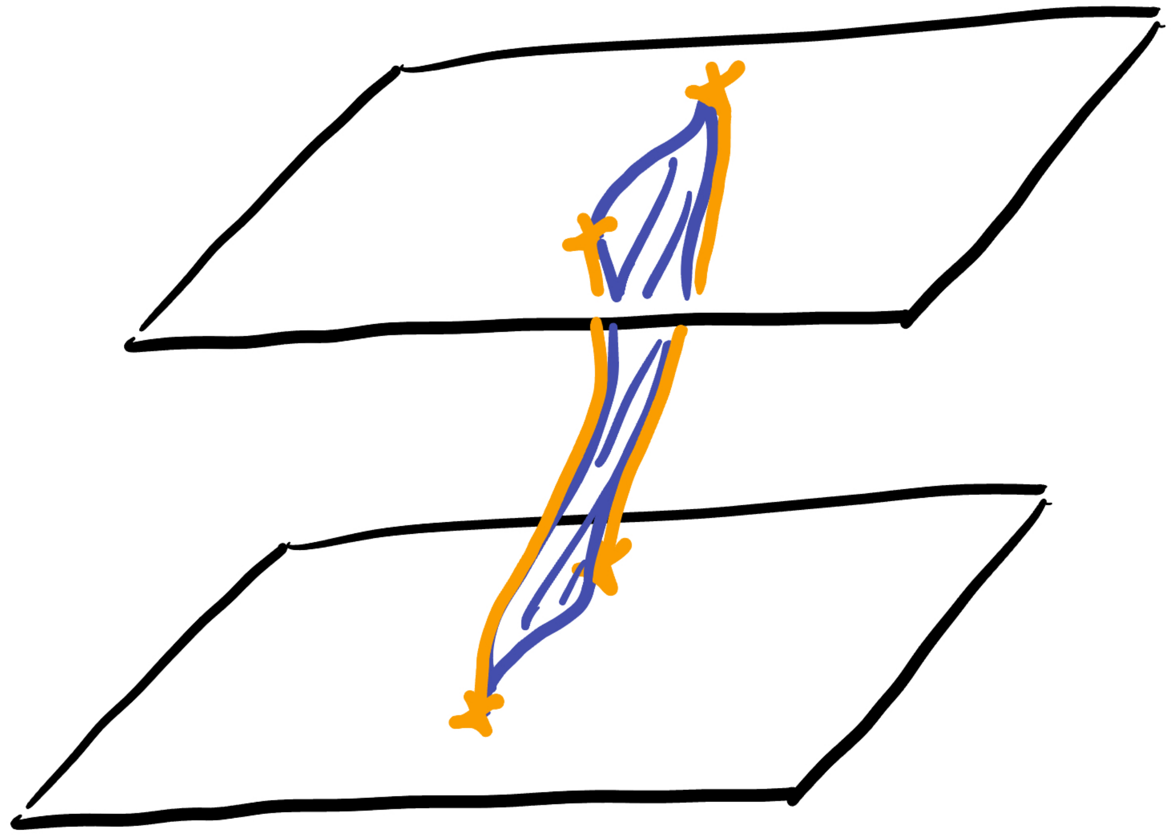
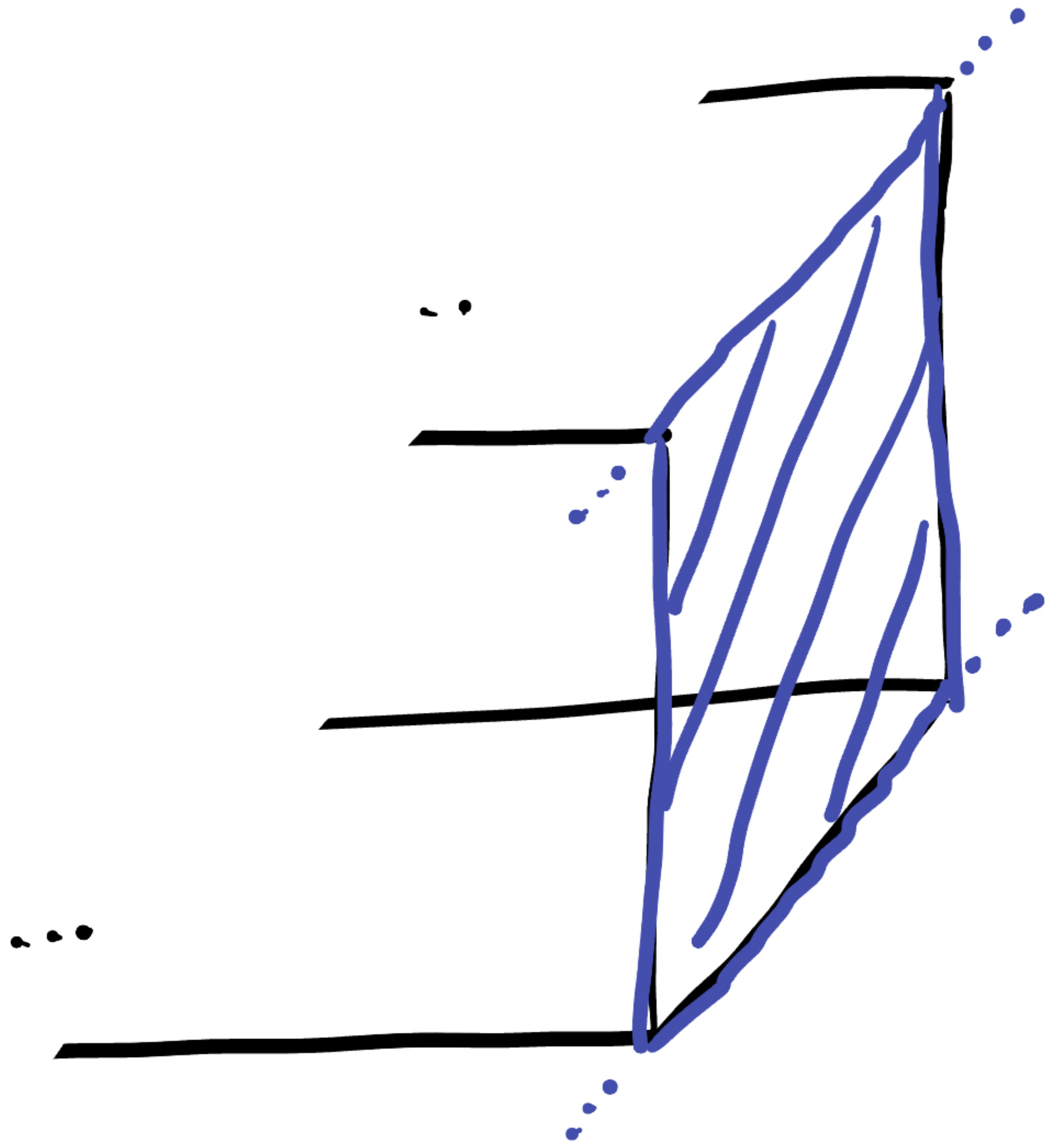


twist defects

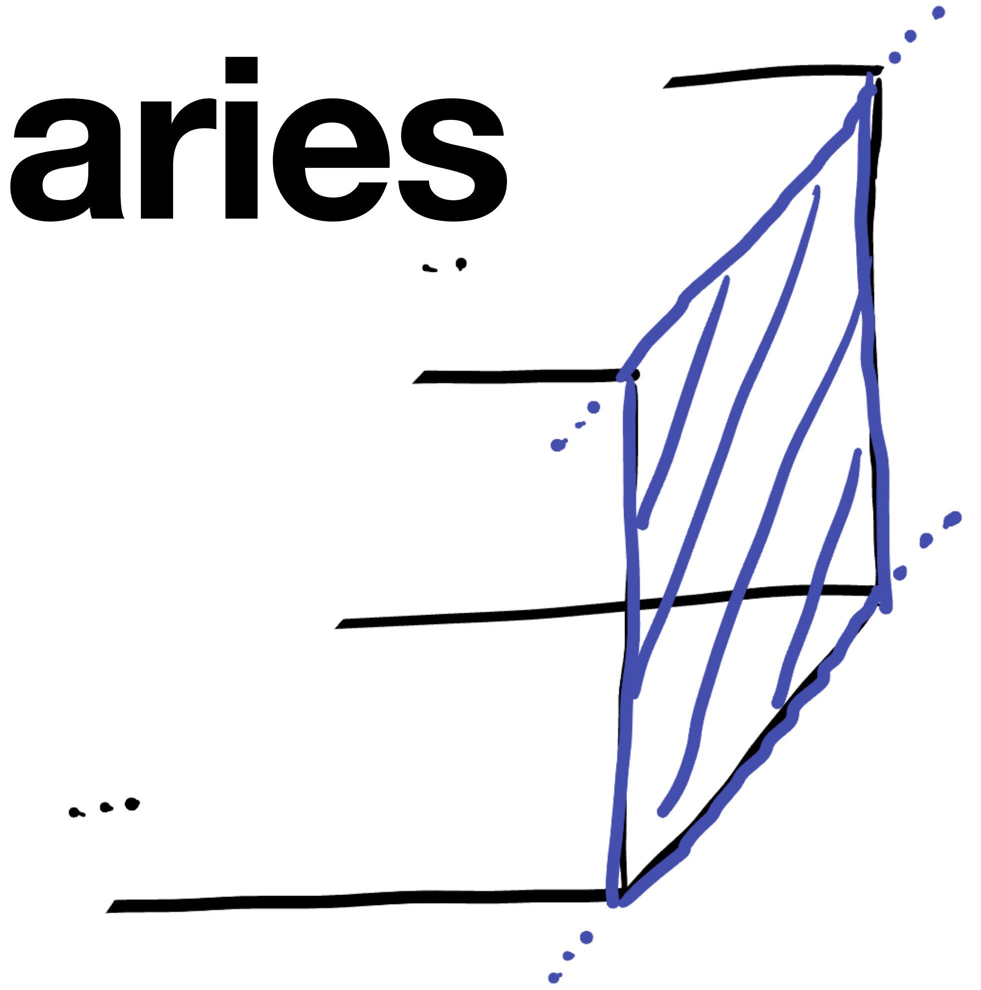
2+1d topological theories with boundaries

“no non-trivial anyons”

boundary: 1+1d **interface to trivial theory**



2+1d topological theories with boundaries



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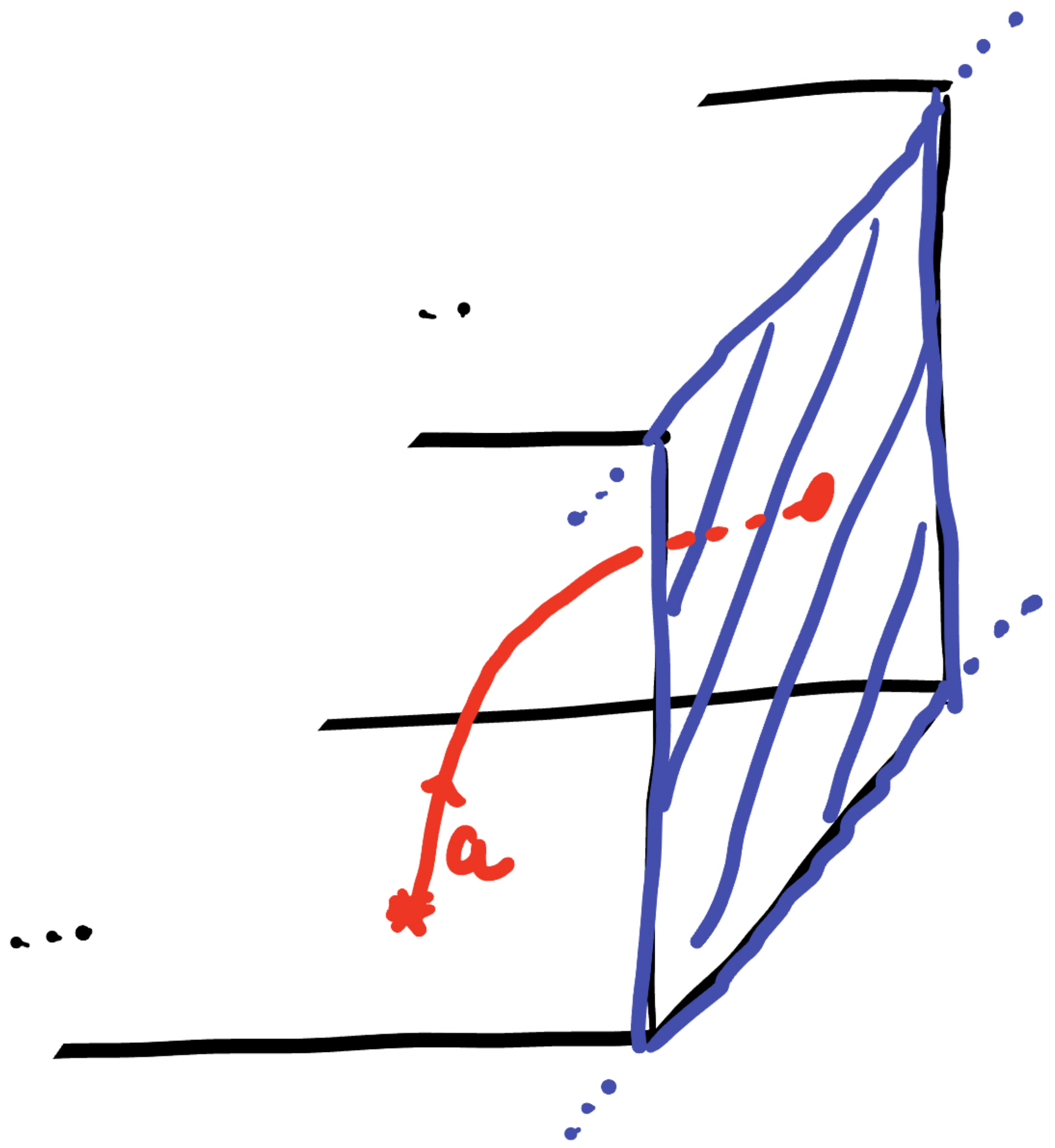
anyons close to the boundary can…



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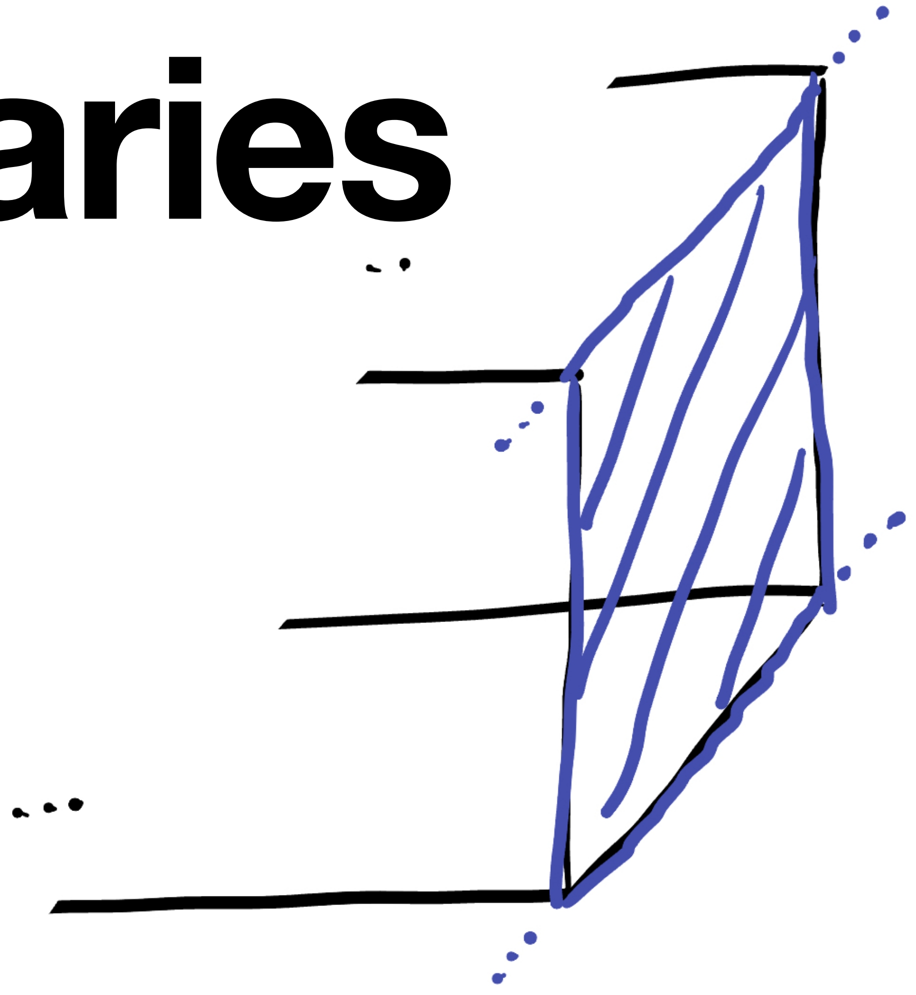
2+1d topological theories with boundaries



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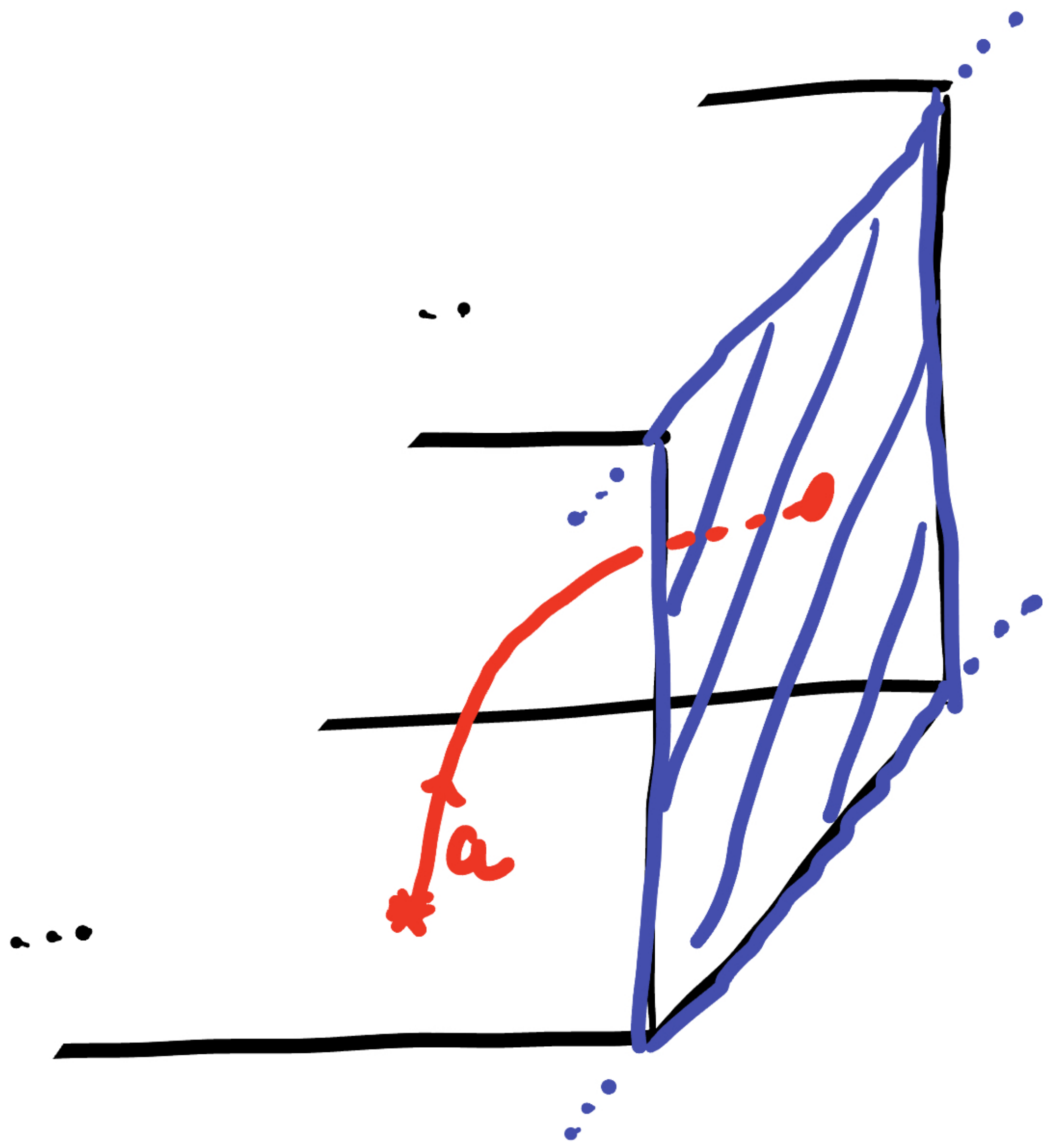

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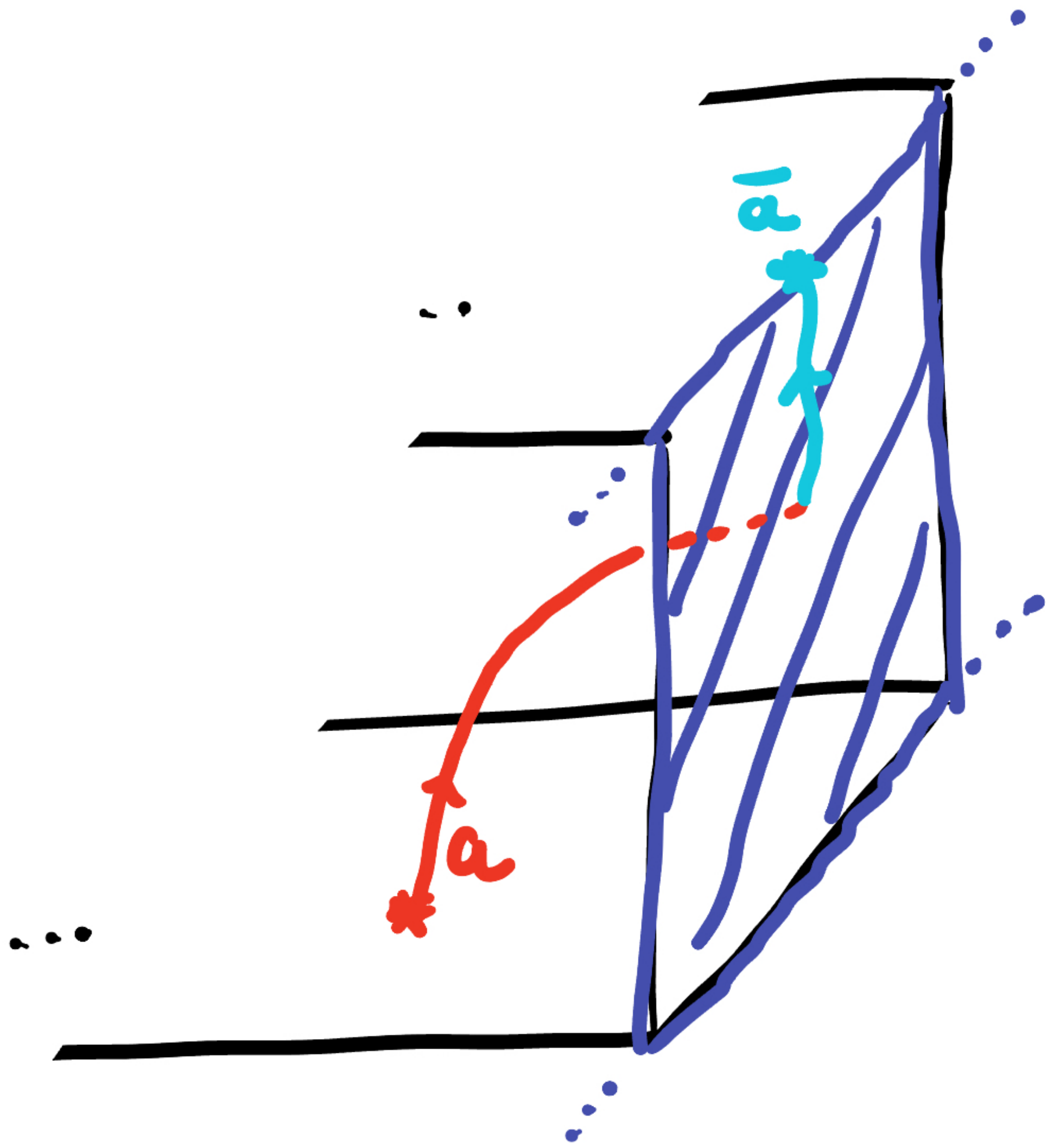
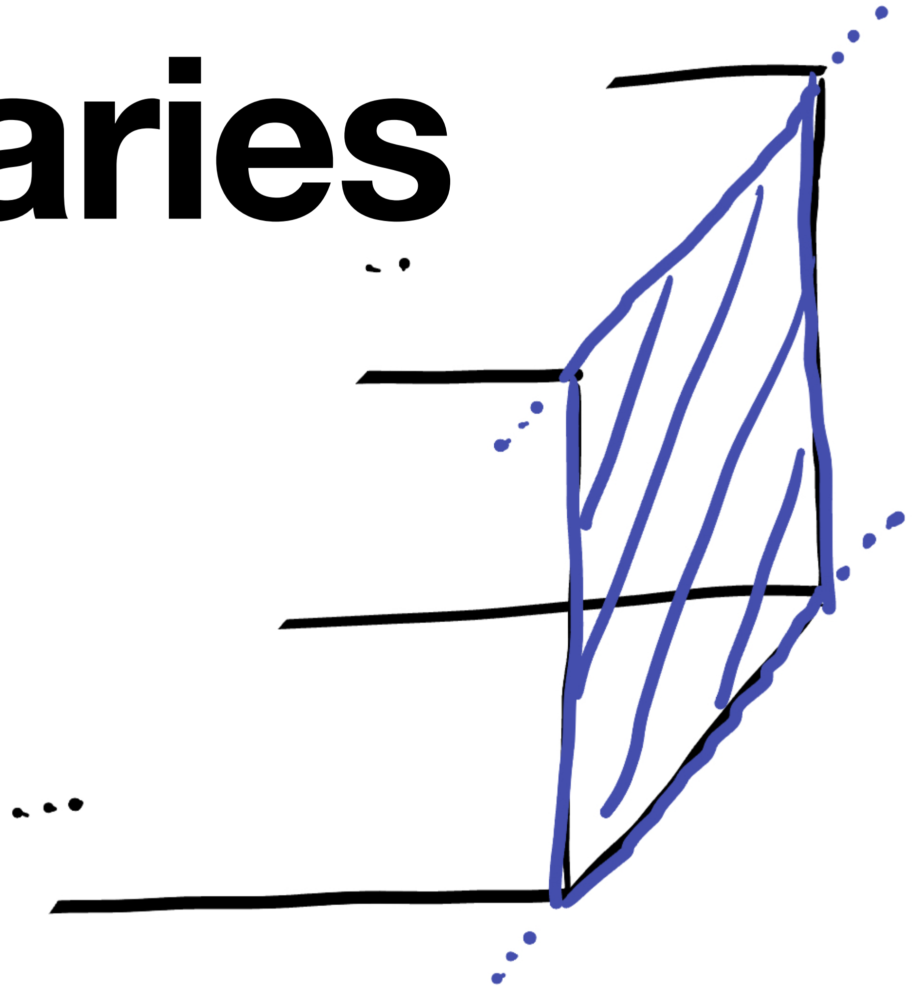


…confine

2+1d topological theories with boundaries



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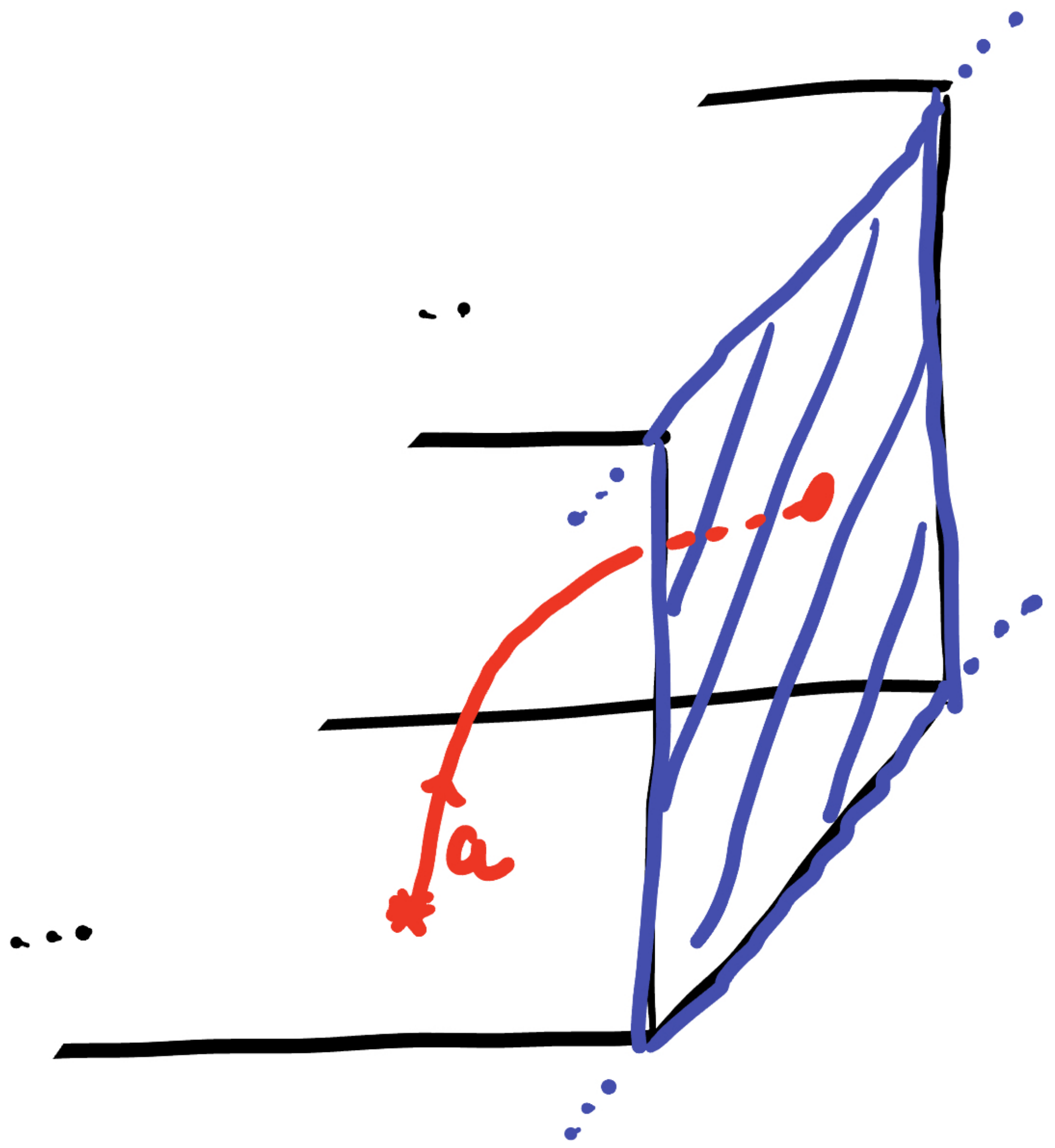

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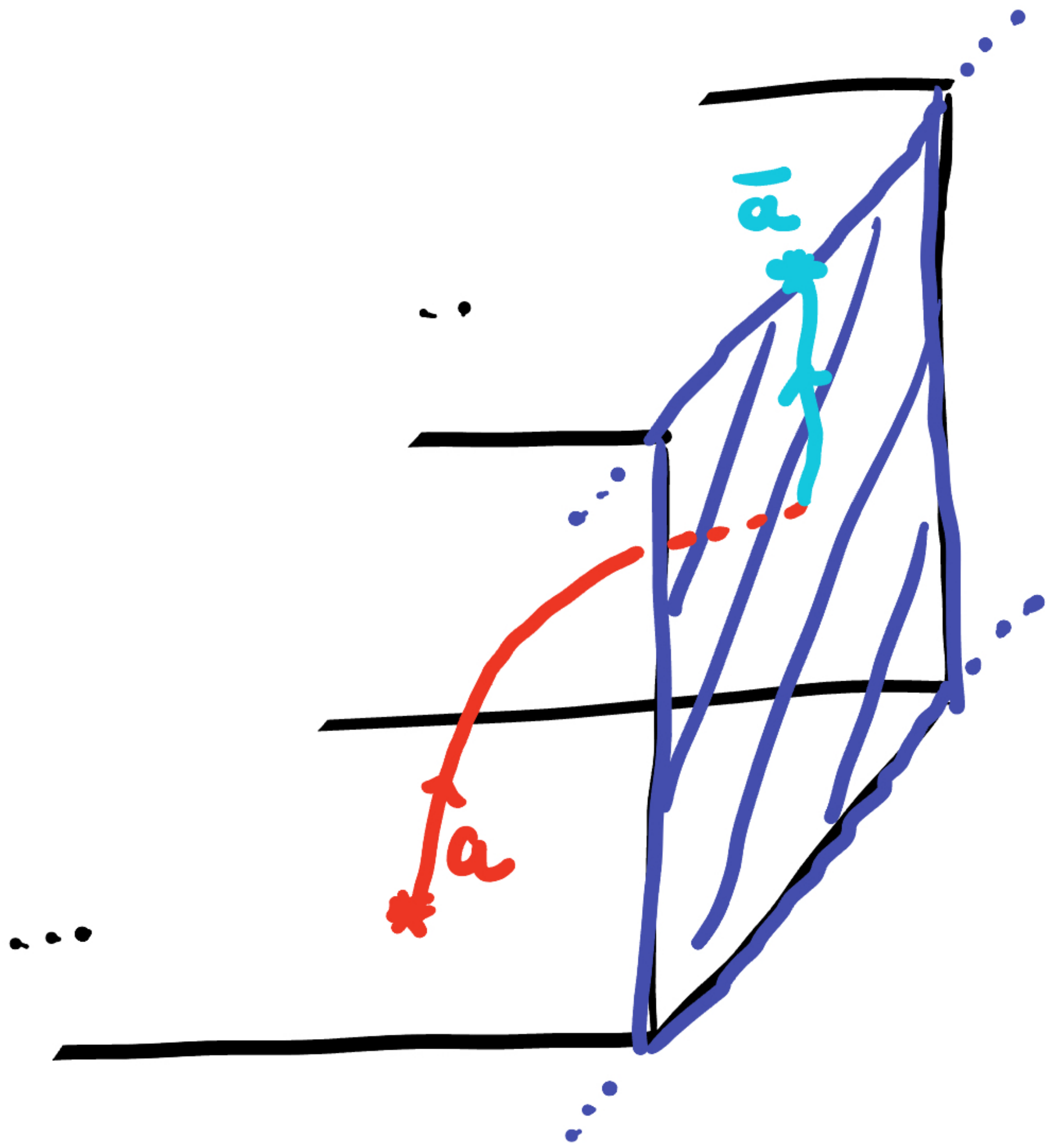

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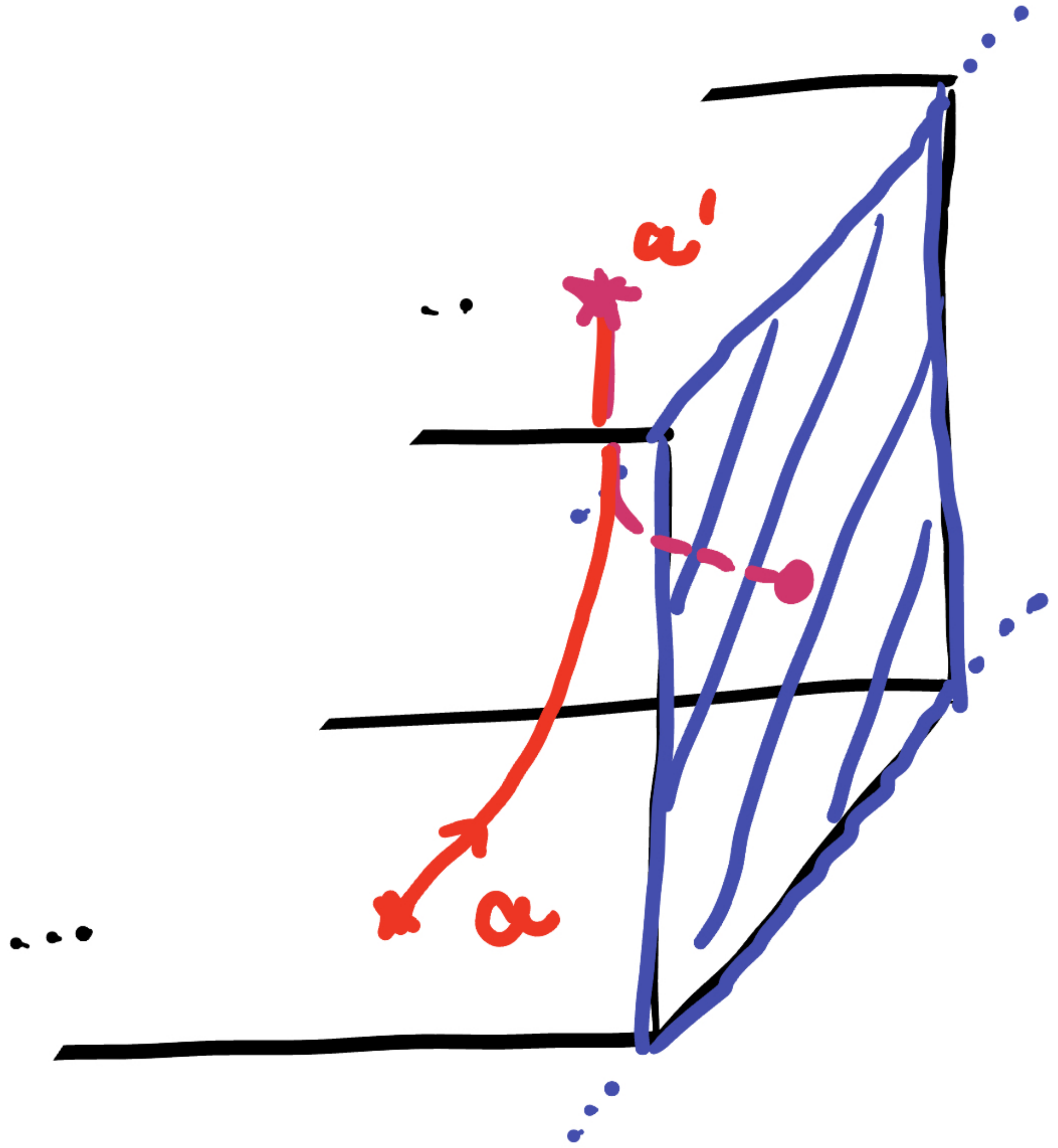

…become identified

folding trick: domain walls as boundaries

domain wall: 1+1d **interface between two** (non-trivial) **theories**

anyon model A $\rightleftharpoons$ anyon model B

Kitaev Kong 1104.5047

folding trick: domain walls as boundaries

domain wall: 1+1d **interface between two** (non-trivial) **theories**

$$\left\{ \boxed{\text{anyon model } A} \rightleftharpoons \boxed{\text{anyon model } B} \right\} \simeq \left\{ \boxed{\text{anyon model } A \boxtimes \overline{B}} \rightleftharpoons \boxed{\text{trivial}} \right\}$$

Kitaev Kong 1104.5047

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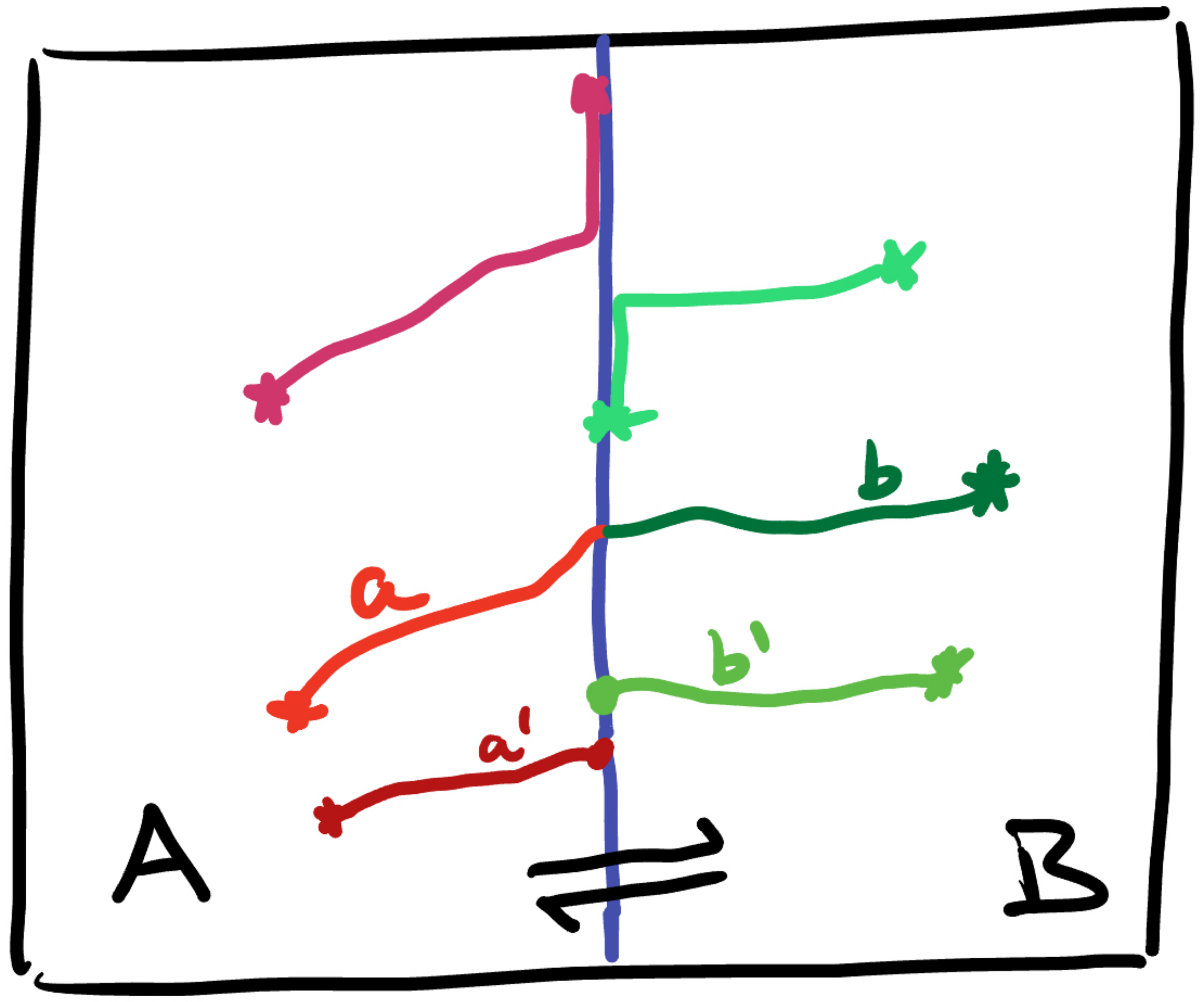
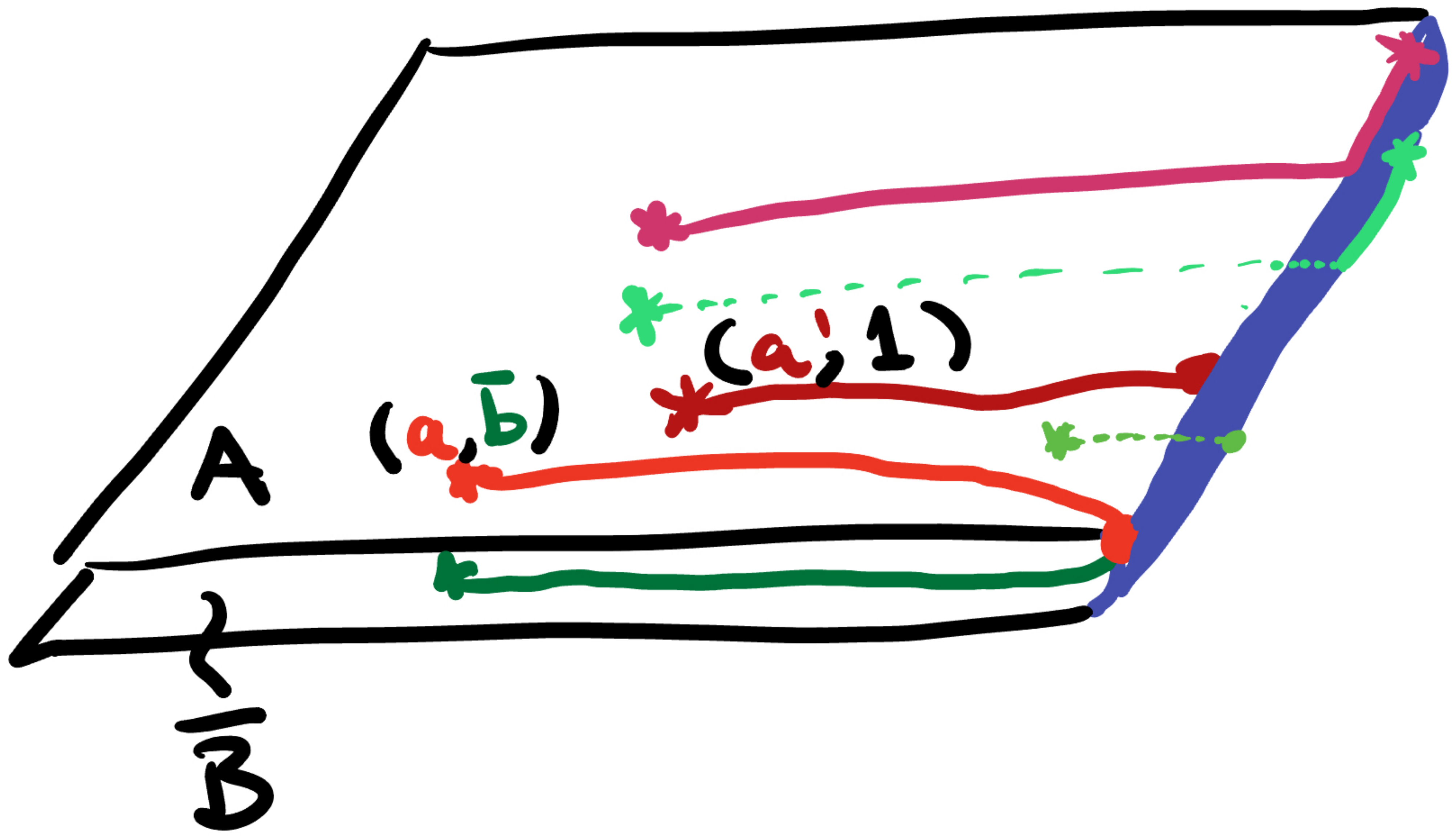
defined by behaviour of anyons close to domain wall



$\simeq$

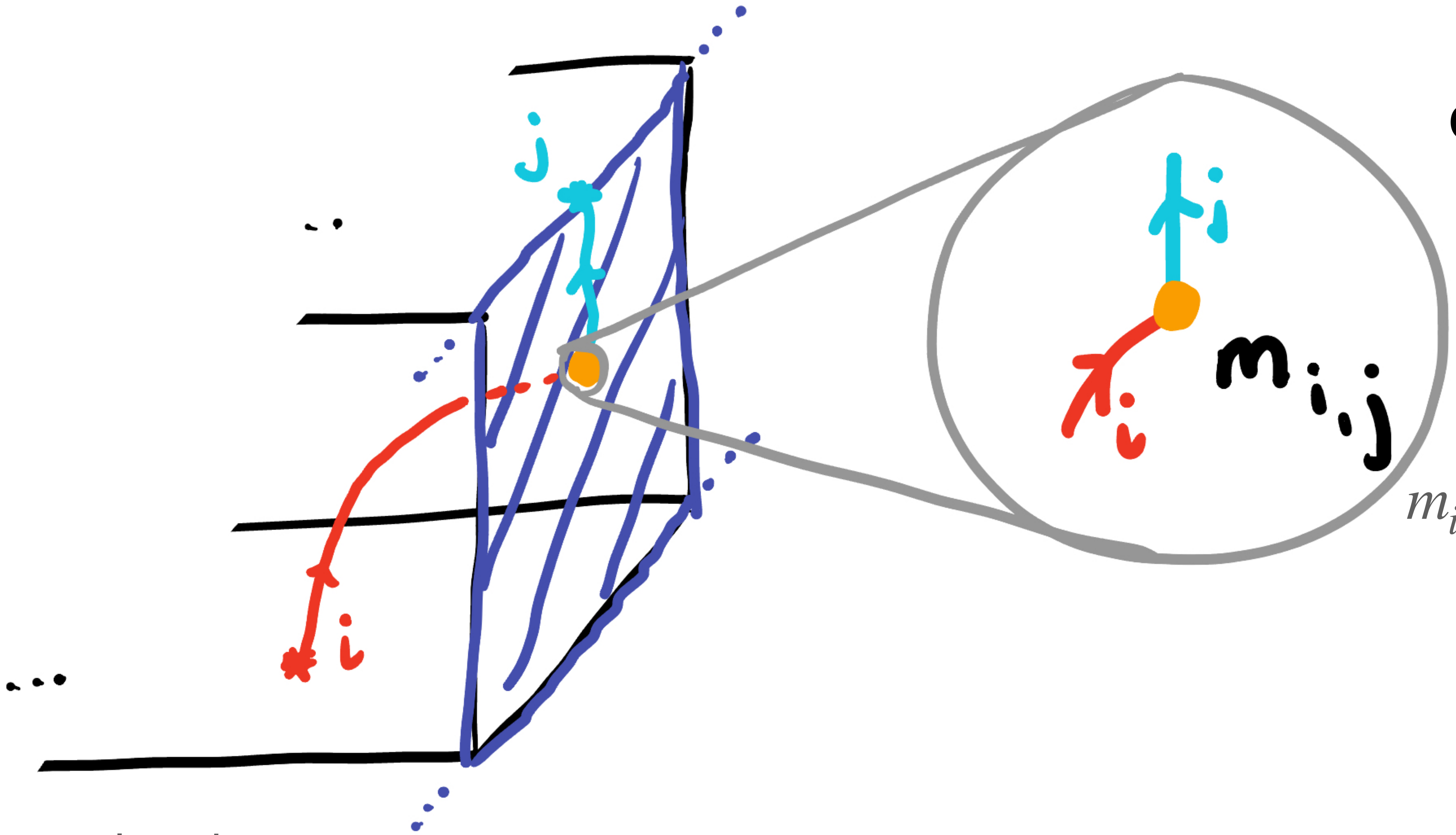


this work: bulk-to-boundary anyon fusion

in non-chiral 2+1D theories

fusion vertex: point (in spacetime) where **bulk anyon i gets mapped to boundary anyon j**

(and vice versa…)



$m_{i,j} \in \mathbb{Z}_{\geq 0}$

defines "tunneling"

$$i \mapsto \bigoplus_j m_{i,j}\ j \quad \text{and} \quad j \mapsto \bigoplus_i m_{i,j}\ i$$

related to:
Carqueville, Runkel, Schaumann 1710.10214
Bridgeman Barter Jones 1806.01279, 1810.09469 and
Bridgeman Barter 1901.08069, 1907.06692

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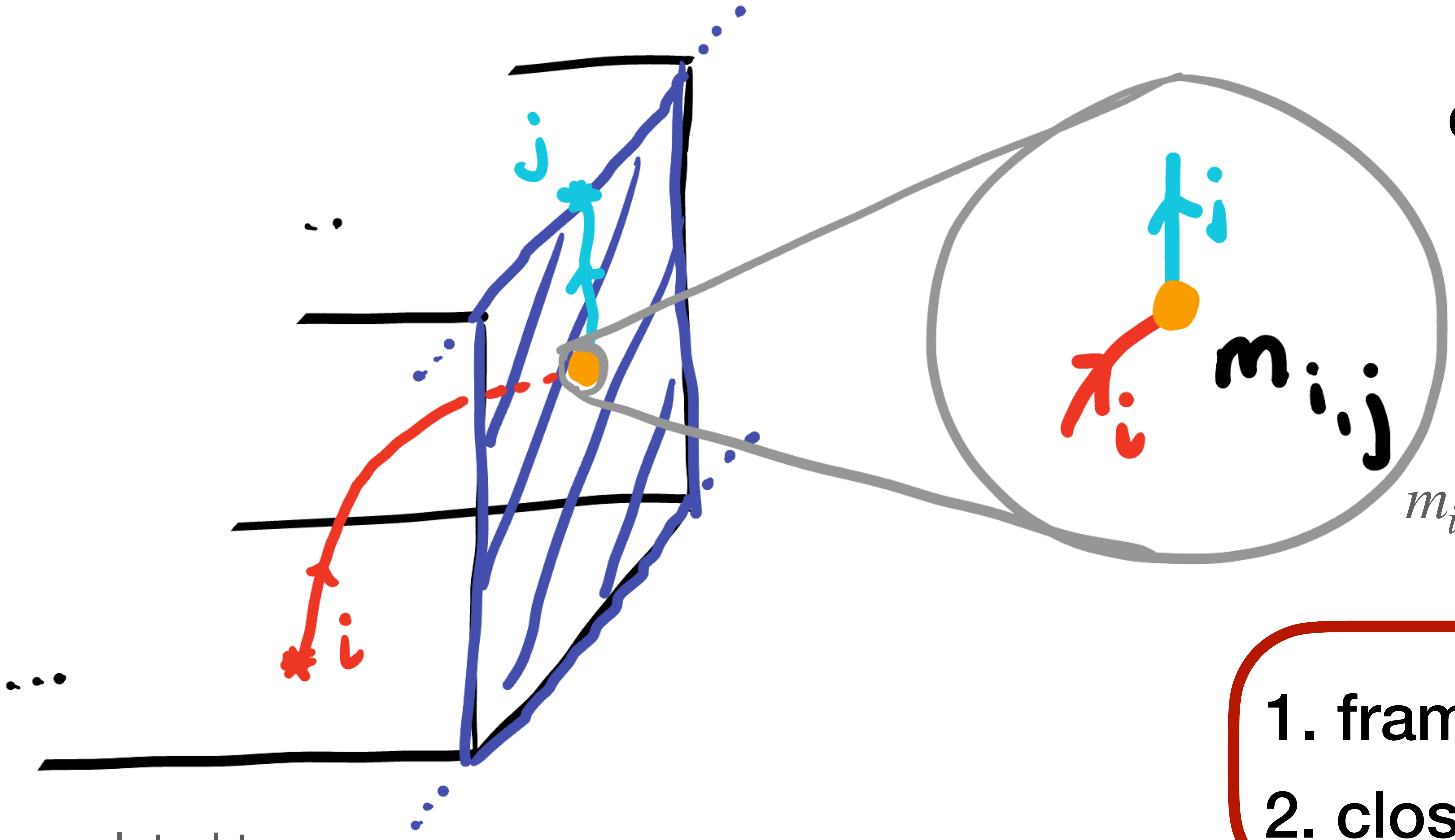

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1. framework to calculate $m_{i,j} \in \mathbb{Z}_{\geq 0}$
2. closed formula for twisted finite gauge theory models

related to:
Carqueville, Runkel, Schaumann 1710.10214
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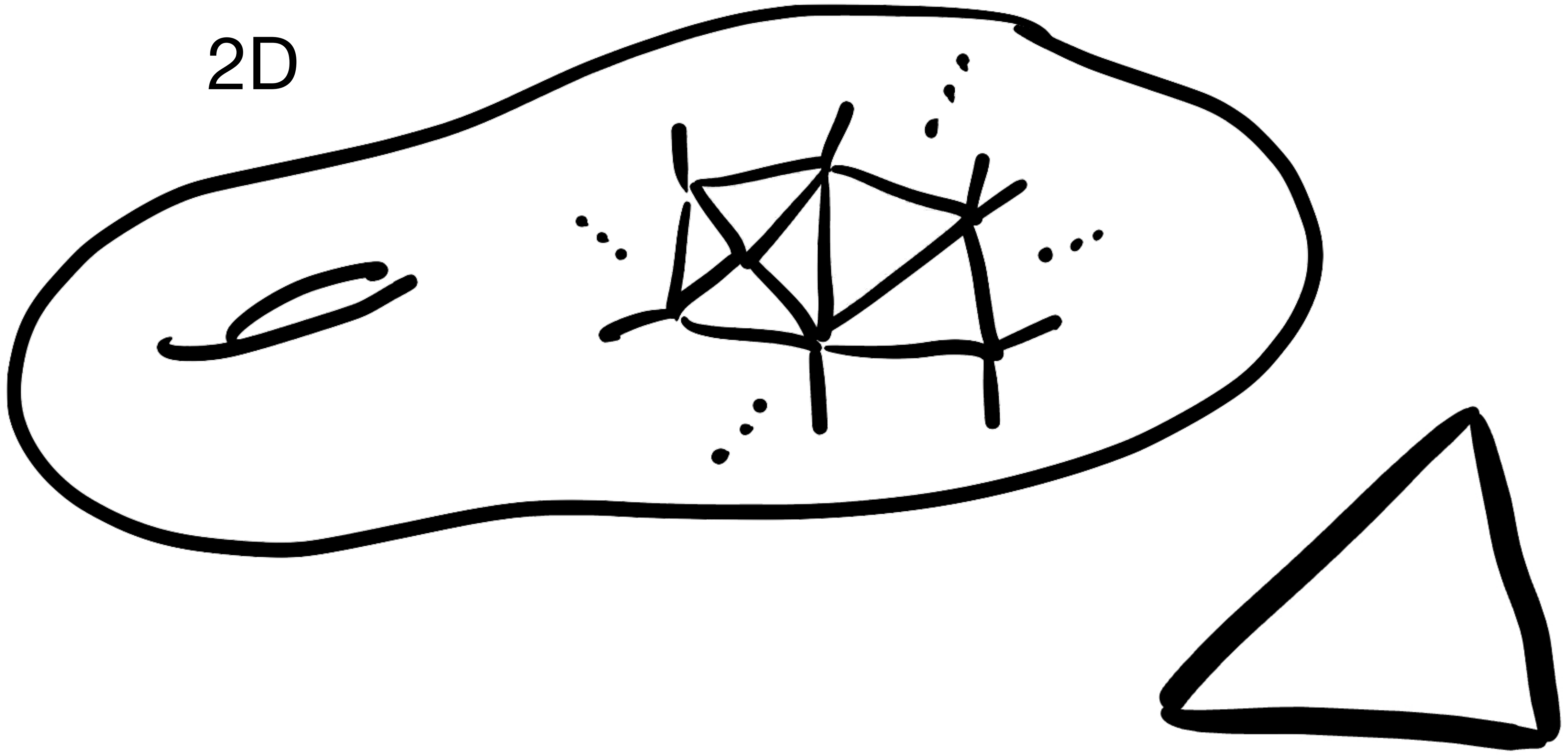
(further) contents

- **microscopic** fixed-point **models** with boundaries
- bulk and boundary **anyons**
- bulk-to-boundary **fusion vertex**
- calculating **fusion multiplicities**
- **applications**
- **conclusion** and outlook

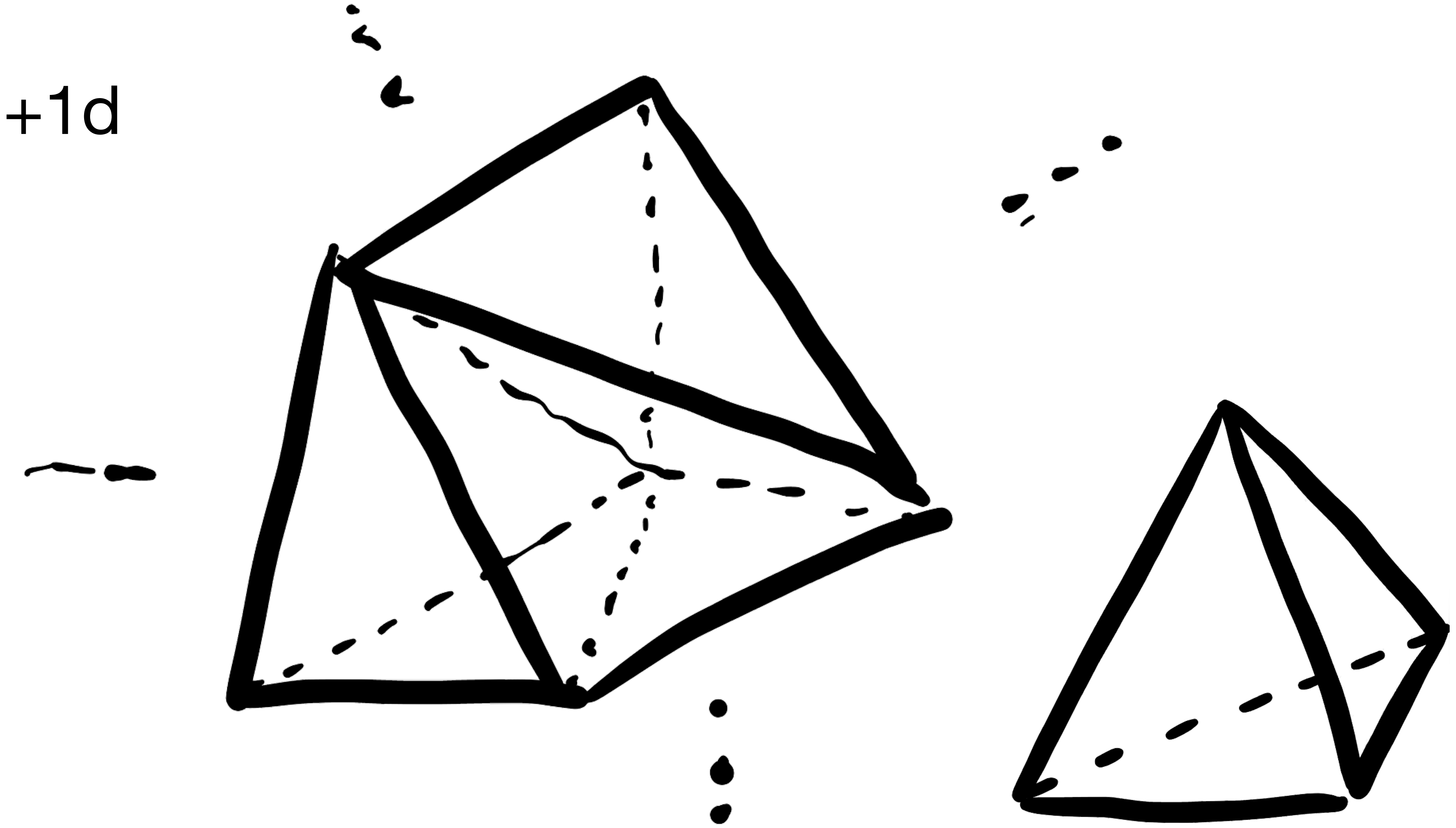
fixed-point models
idea

define theory in terms of **microscopic model with exact topological invariance**

2D



2+1d



fixed-point models

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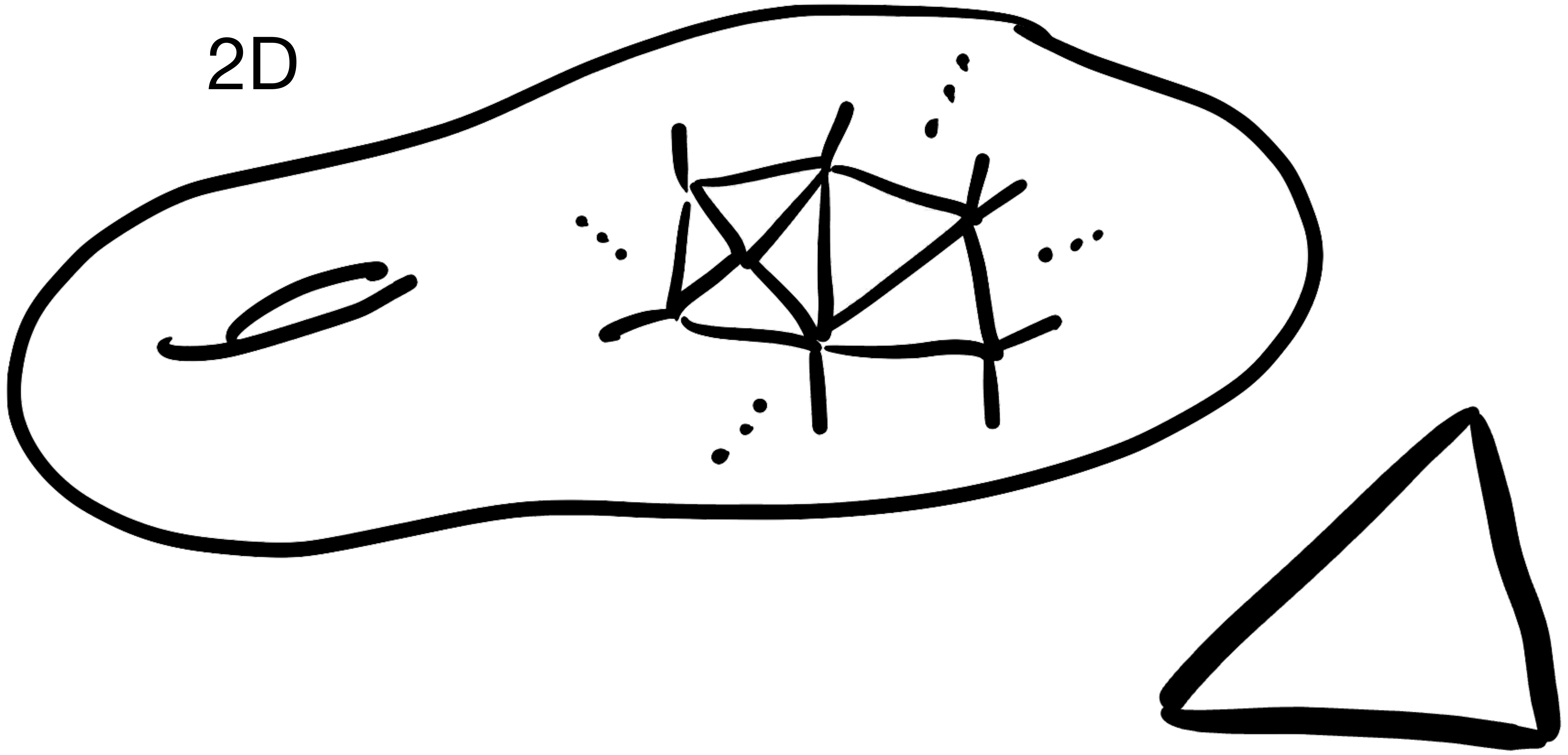
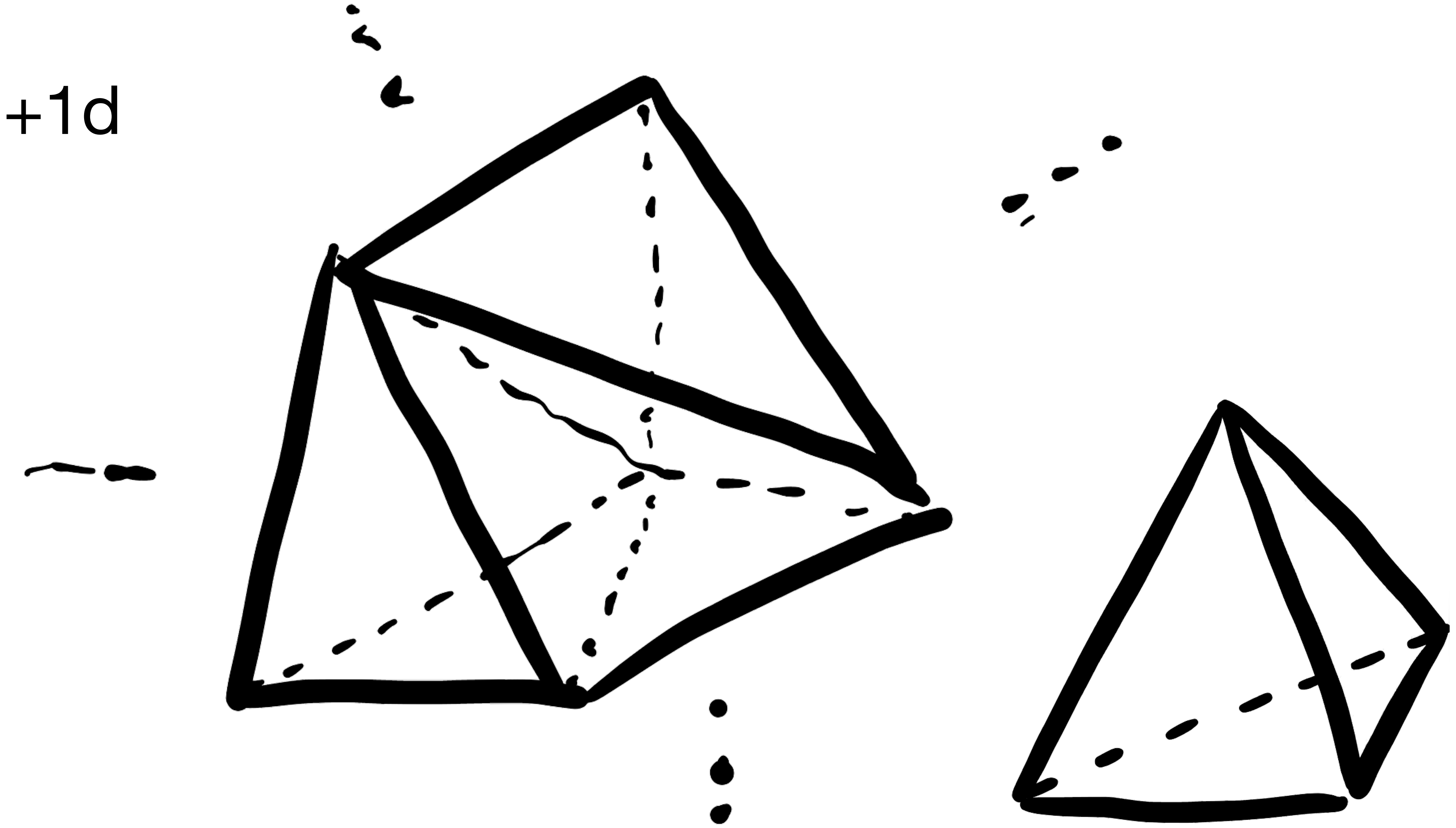
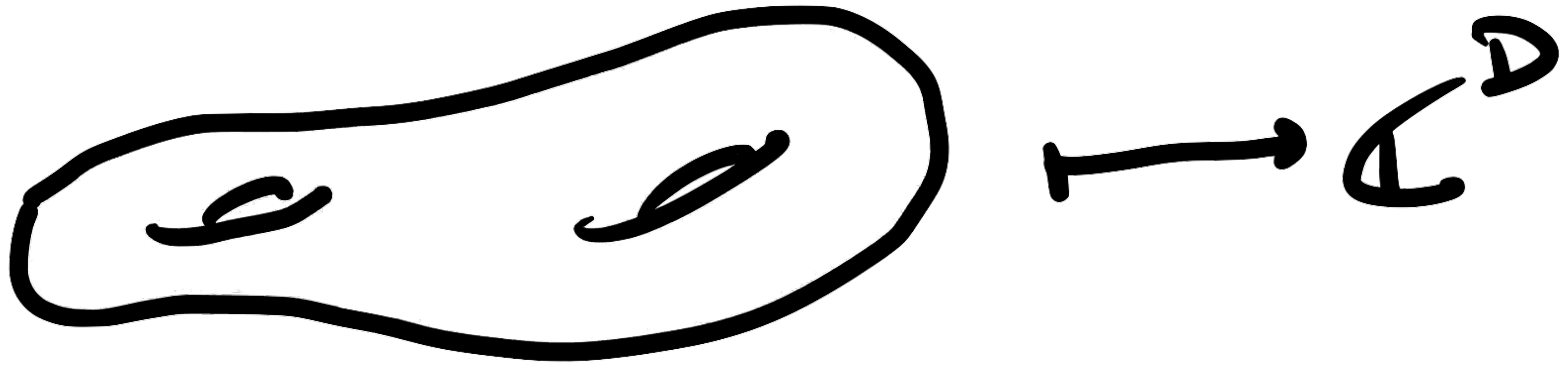




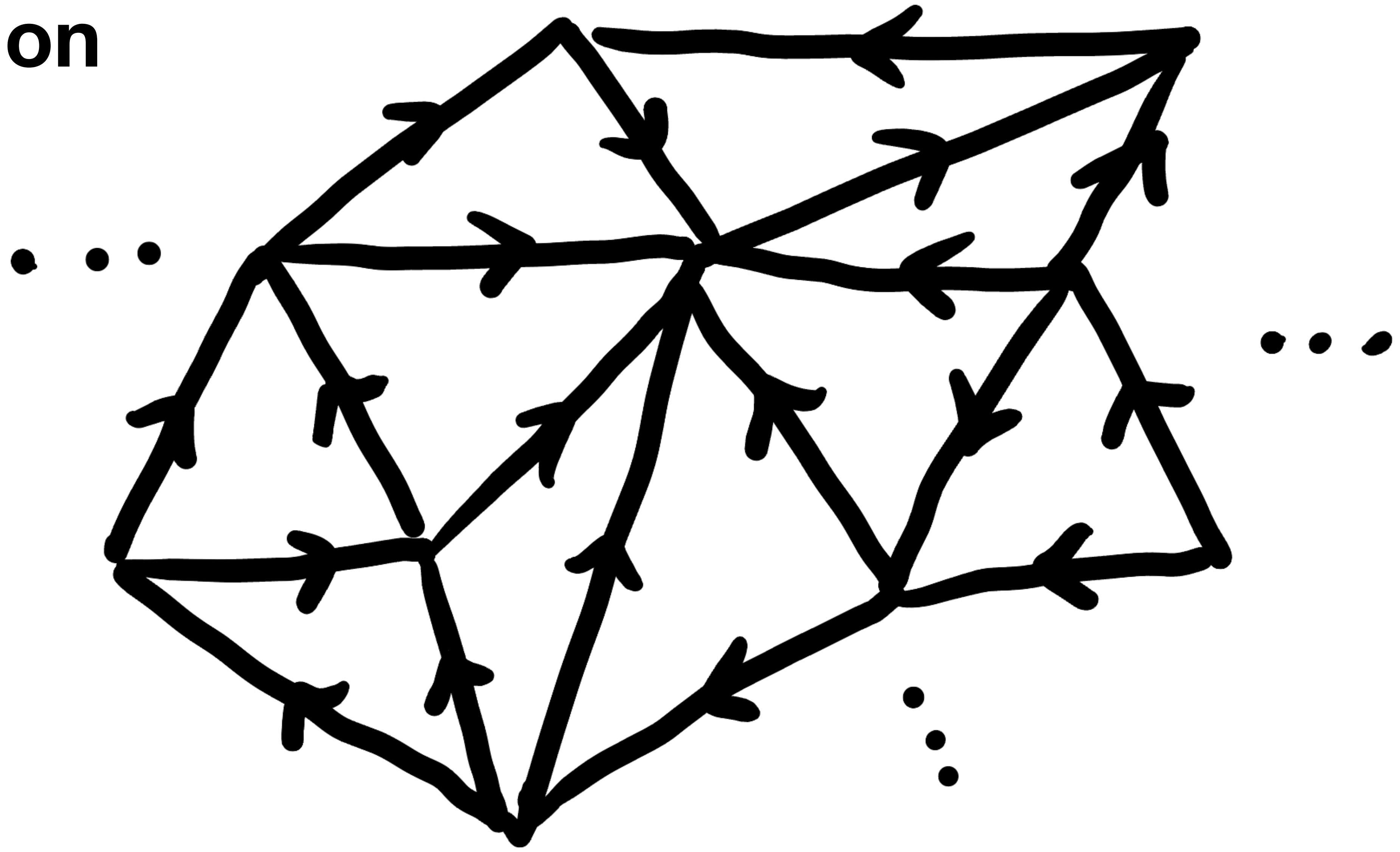
defects implemented by **modifying model along submanifold**

fixed-point models

bulk



2D “string-net” models defined on **branched triangulation**



fixed-point models

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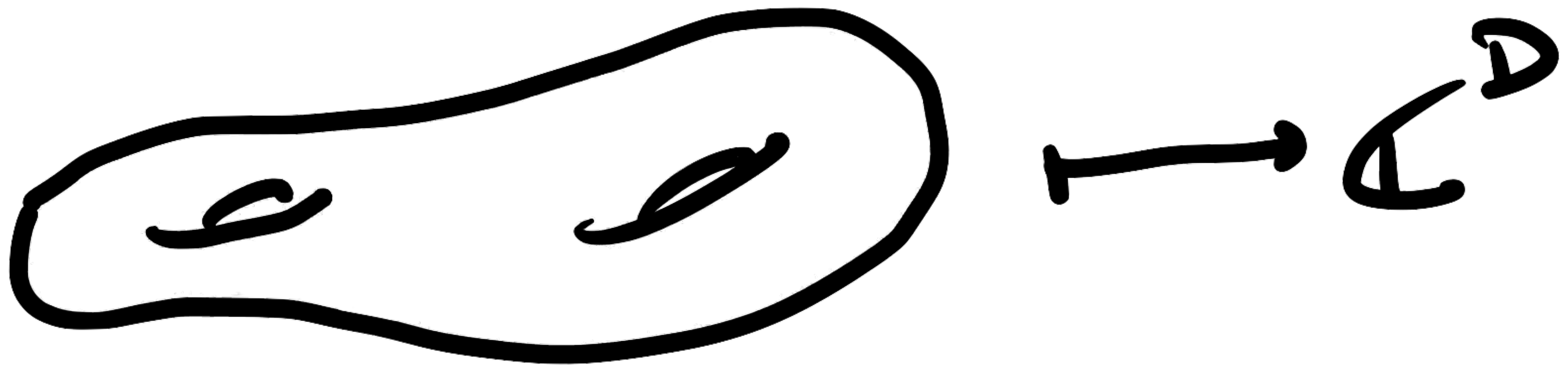

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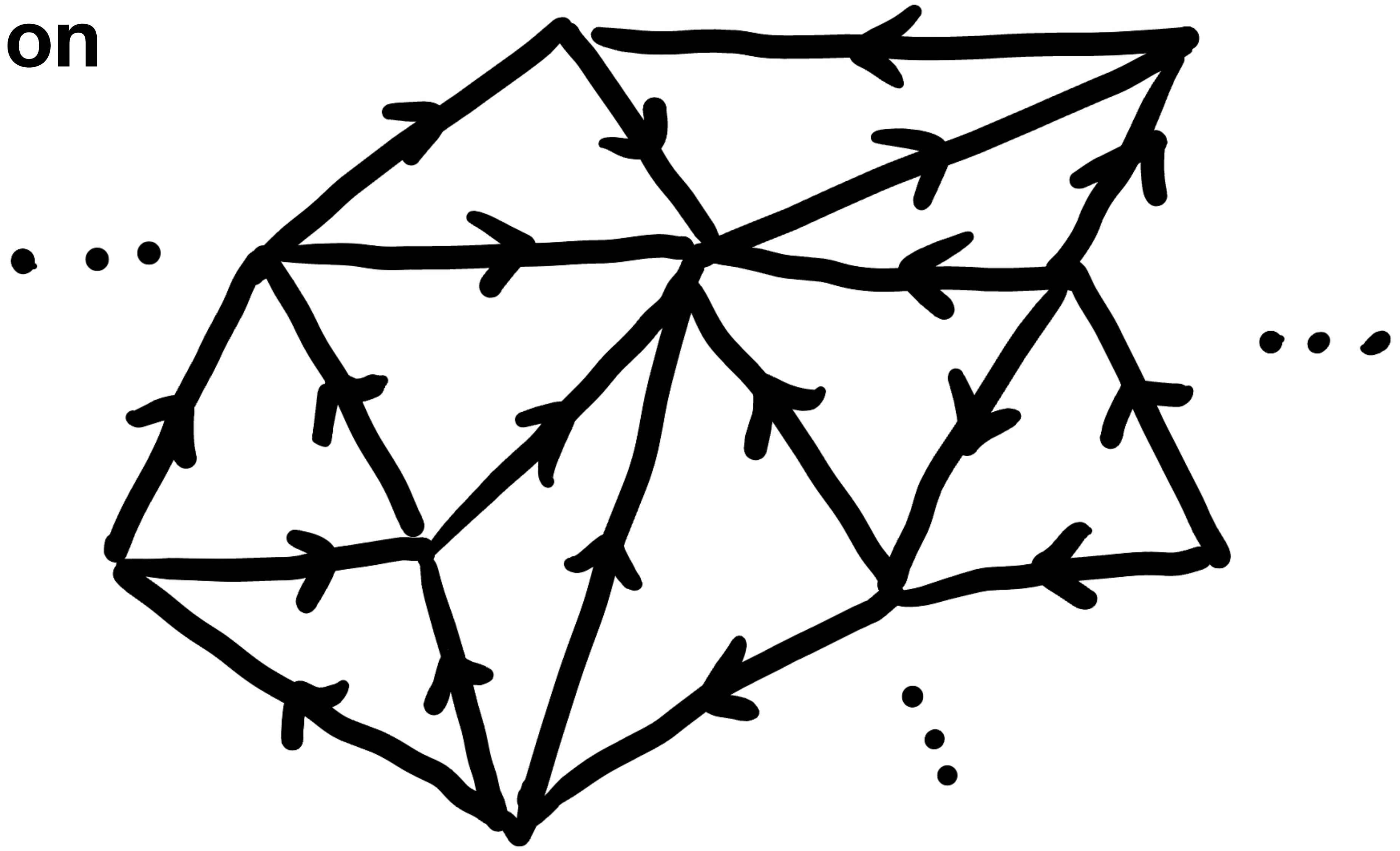


states embedded into **finite-dim. tensor product space**

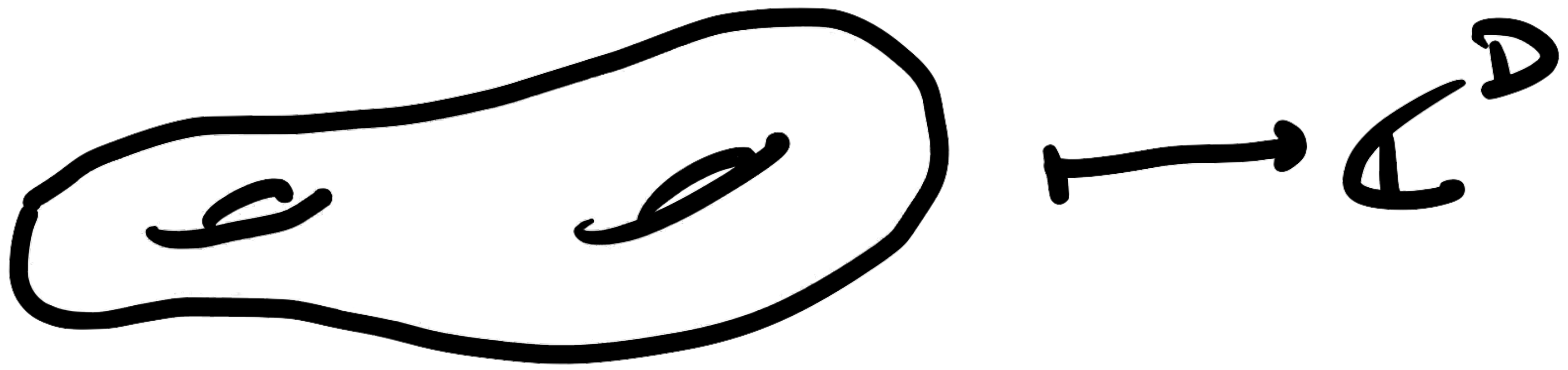
$$\mathscr{H} = \bigotimes_{\text{edges } e} \text{span}_{\mathbb{C}}\{1, i, j, k, \ldots\}$$

with **local constraints** @ each triangle

$$\boxed{i \times j = \bigoplus_k N_{ij}^k \, k, \quad N_{ij}^k \in \mathbb{Z}^+}$$

fixed-point models

bulk



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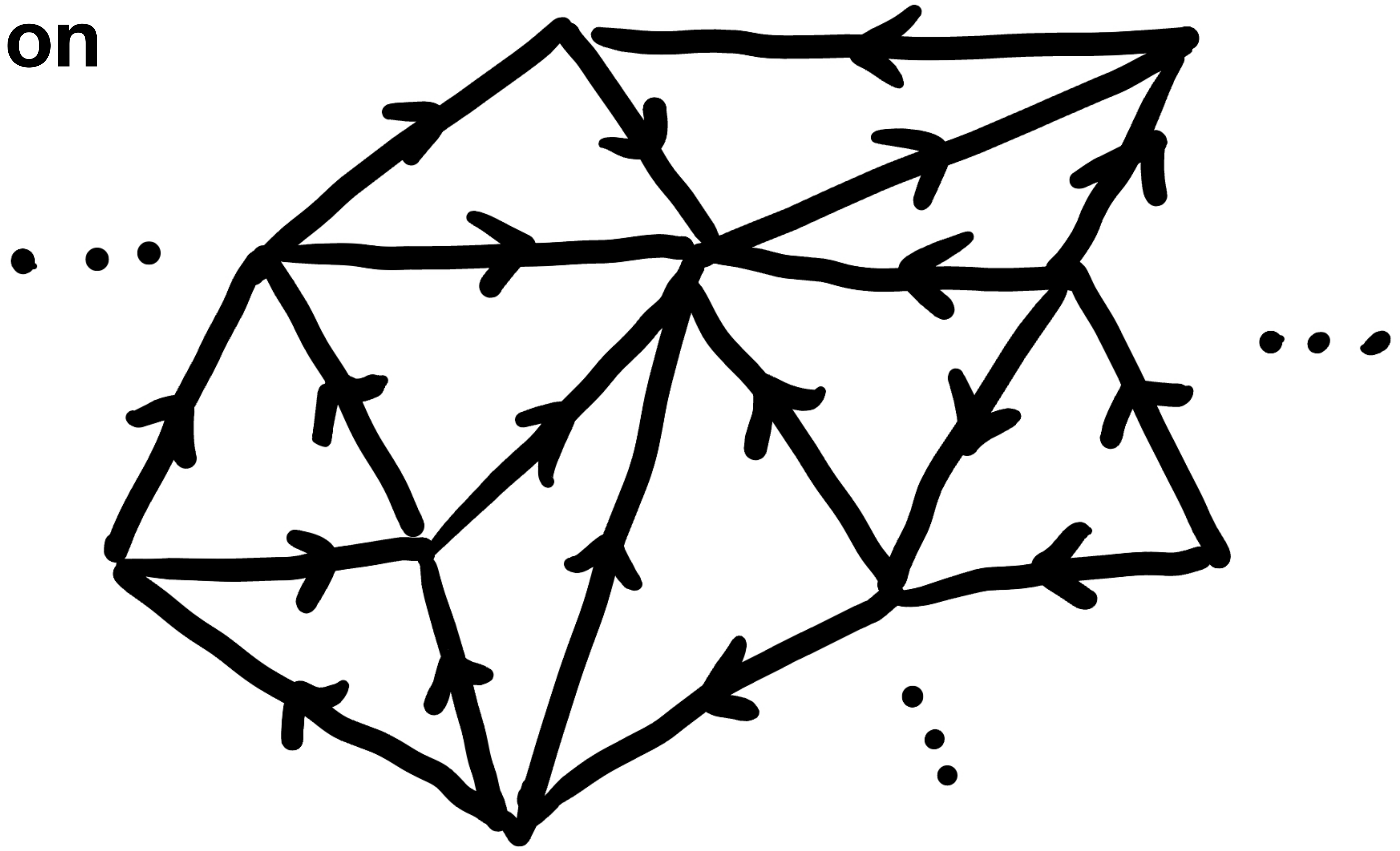

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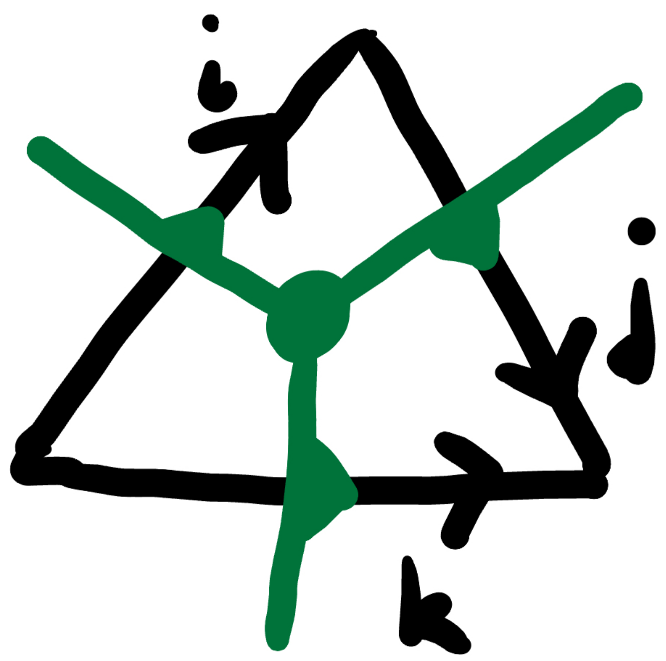
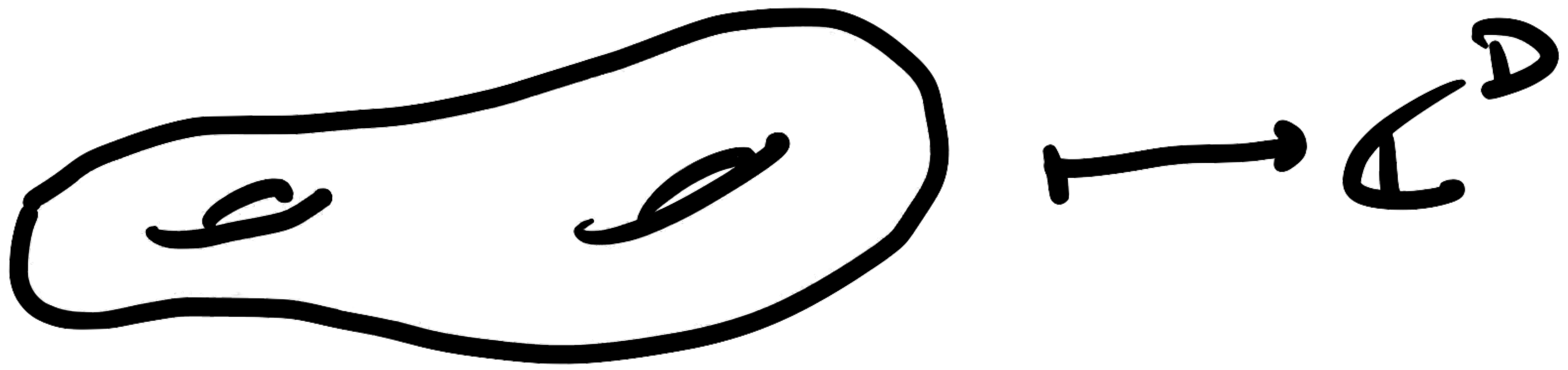


$\hat{=}\ N_{ij}^k$

fixed-point models

bulk



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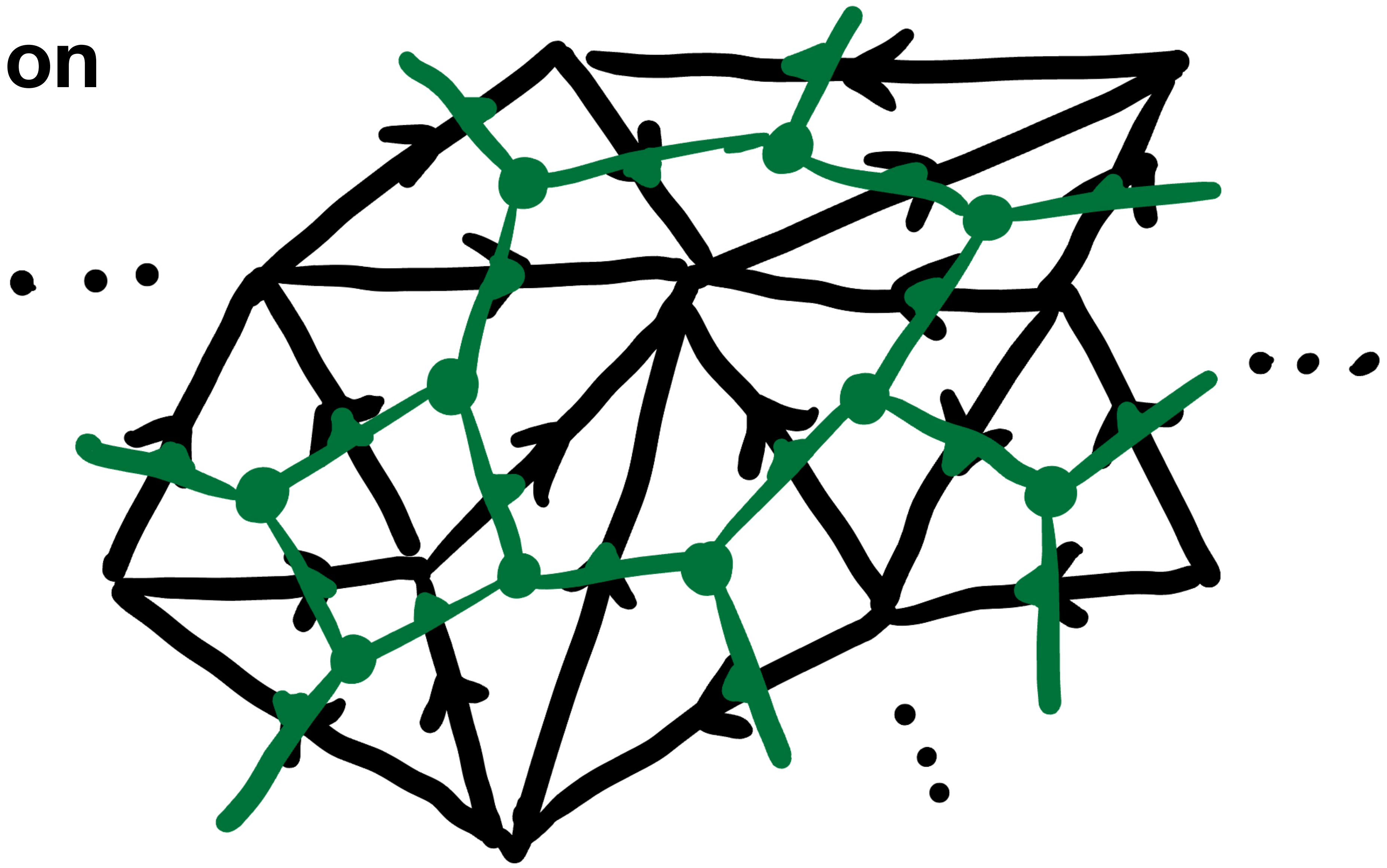

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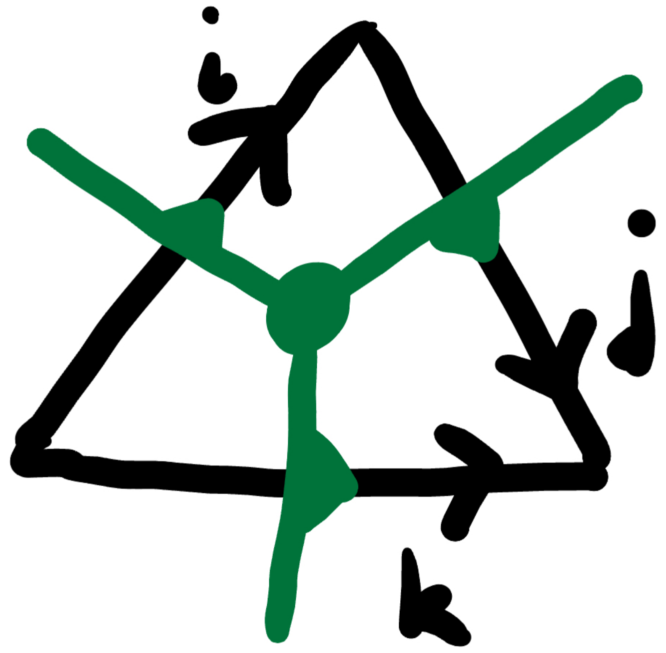

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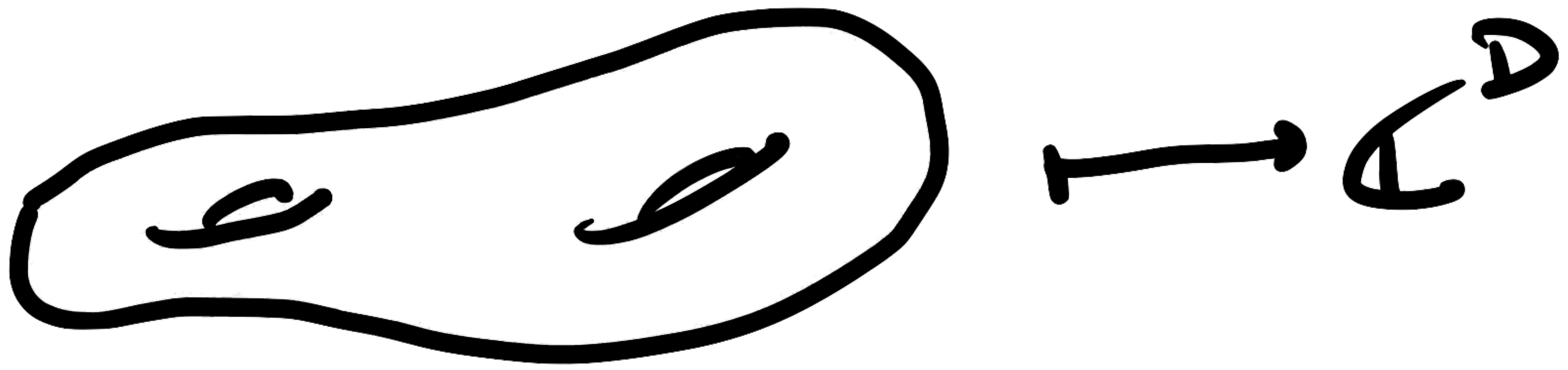
fixed-point models

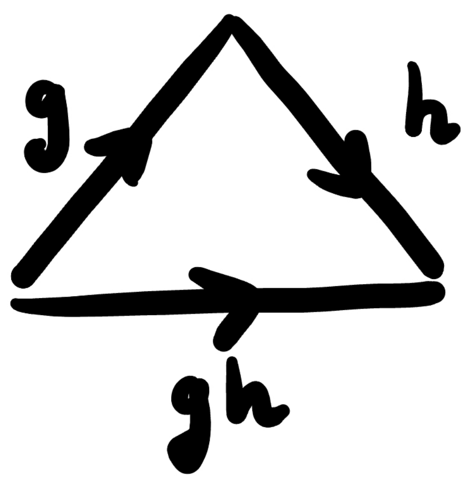
bulk



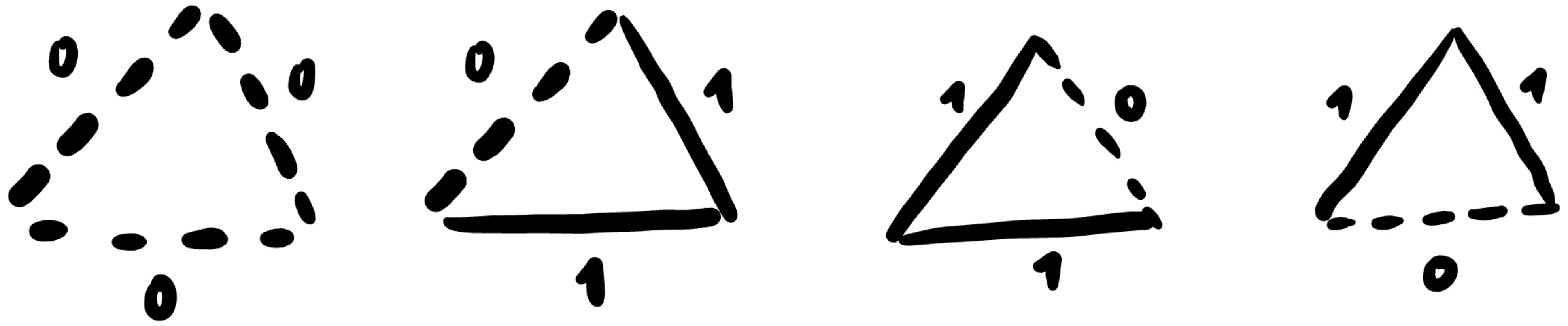
Example 1

finite group $G = \{1, g, h, k, \dots\}$ defines

$$g \times h = gh \quad \Longrightarrow \quad N^k_{g,h} = \delta_{k,gh}$$

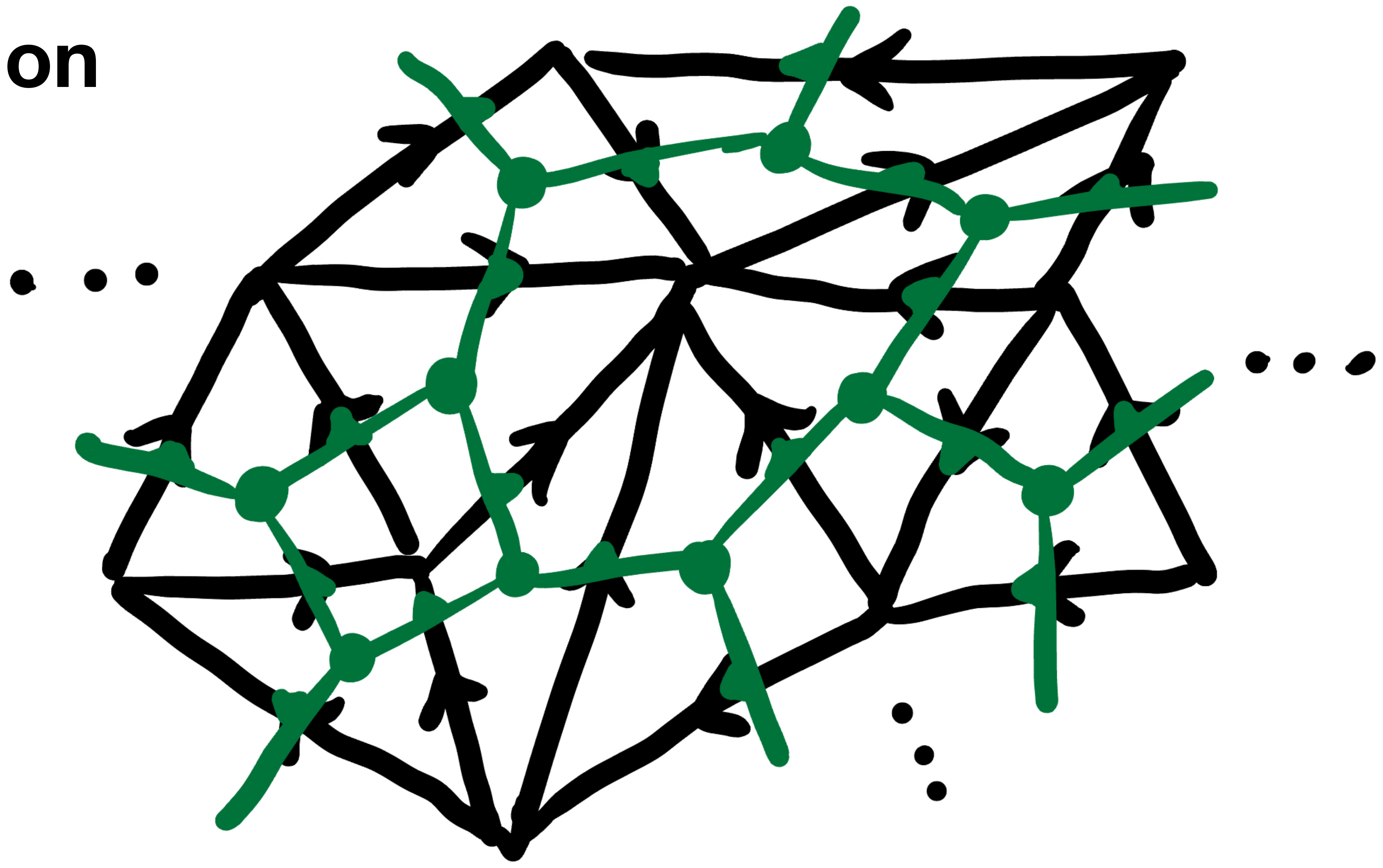


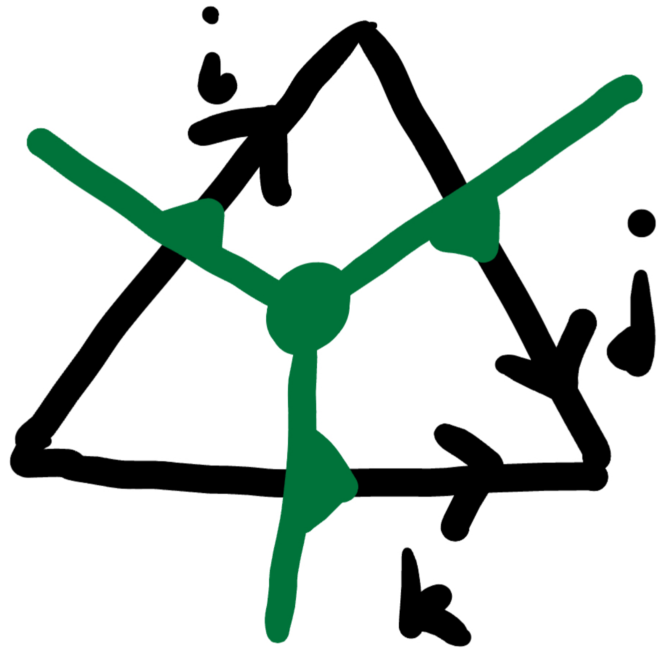
e.g. $G = \mathbb{Z}_2 = \{0,1\}$:



riangulation

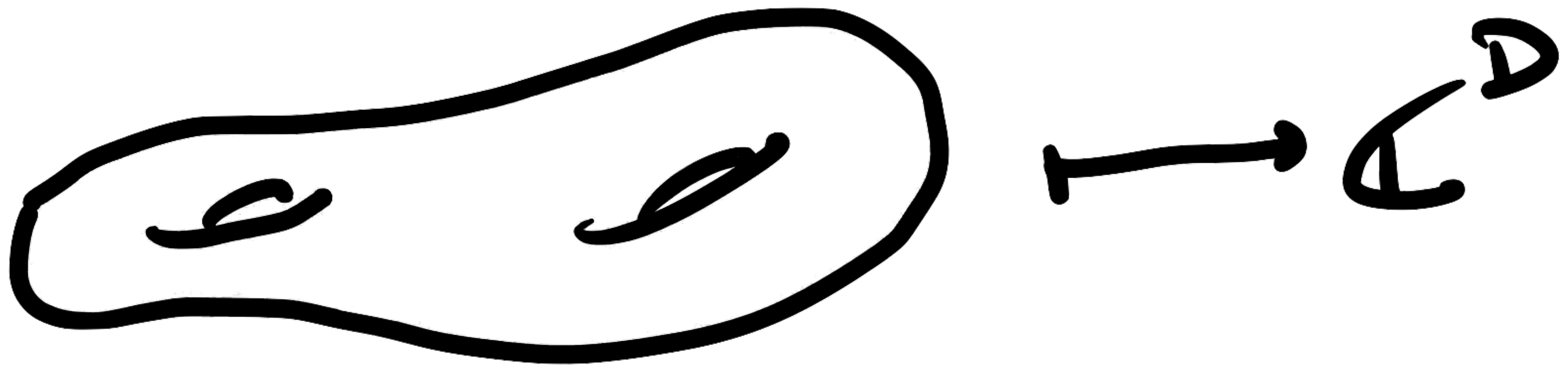
ce





$\hat{=}\ N^k_{ij}$

fixed-point models



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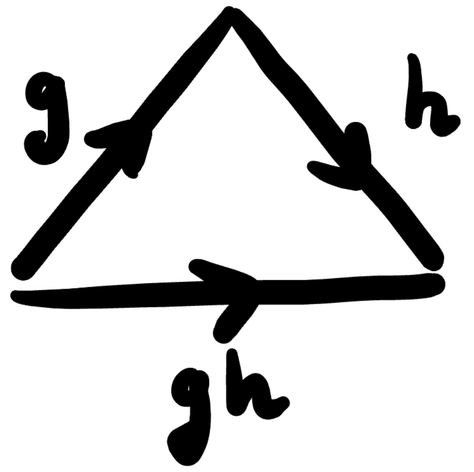

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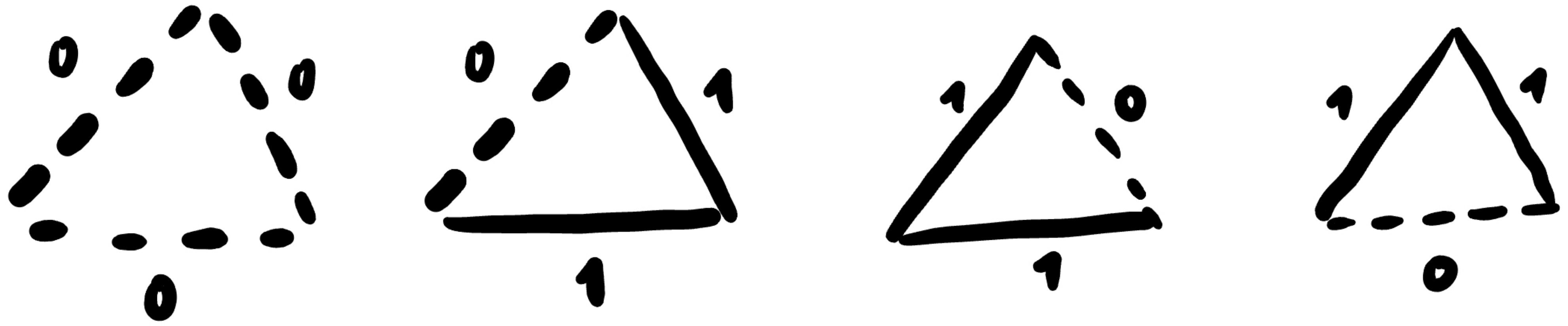


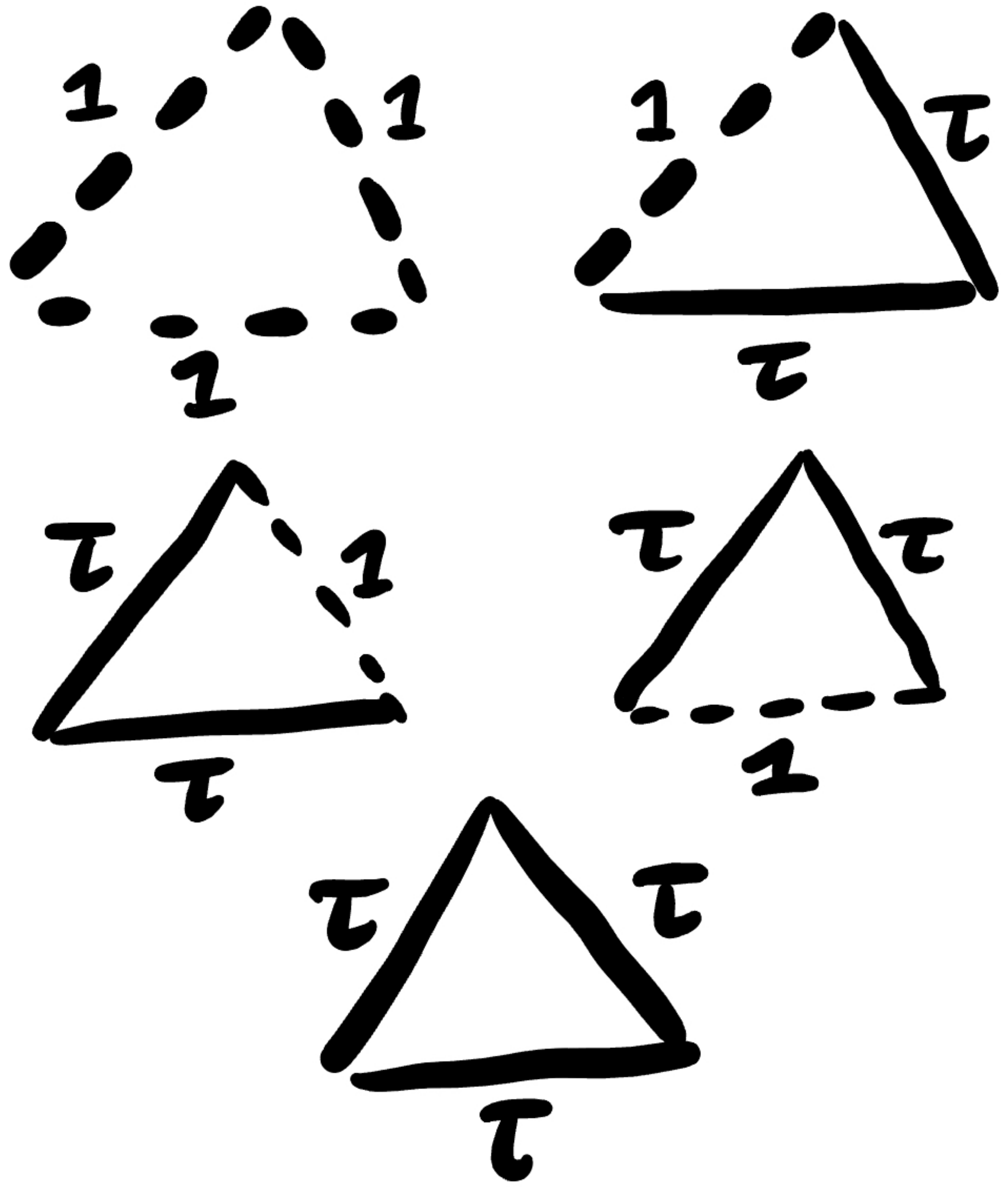
rian

ce

Example 2

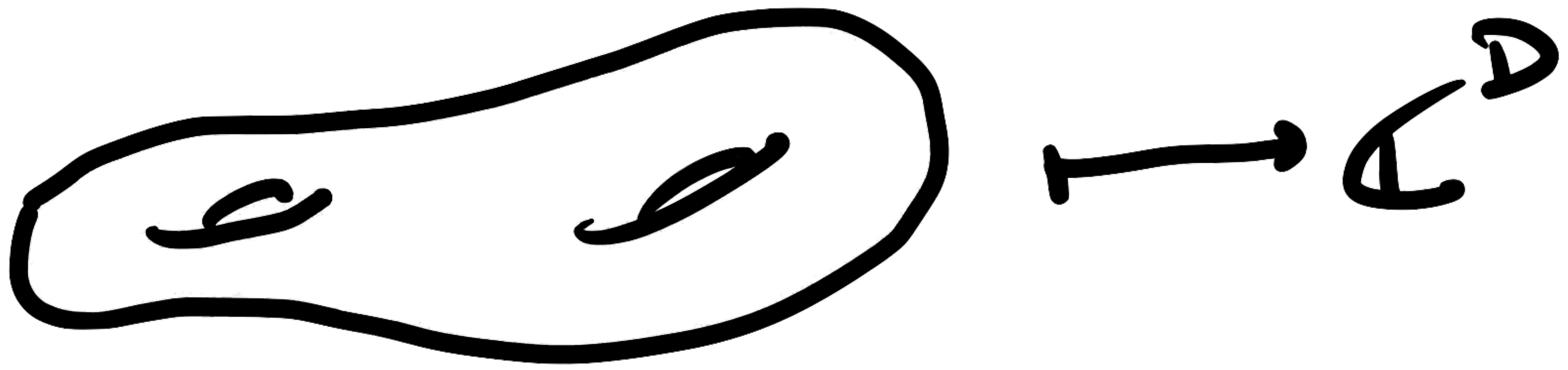
beyond group: $\{1, \tau\}$ with

$$\tau \times \tau = 1 + \tau$$



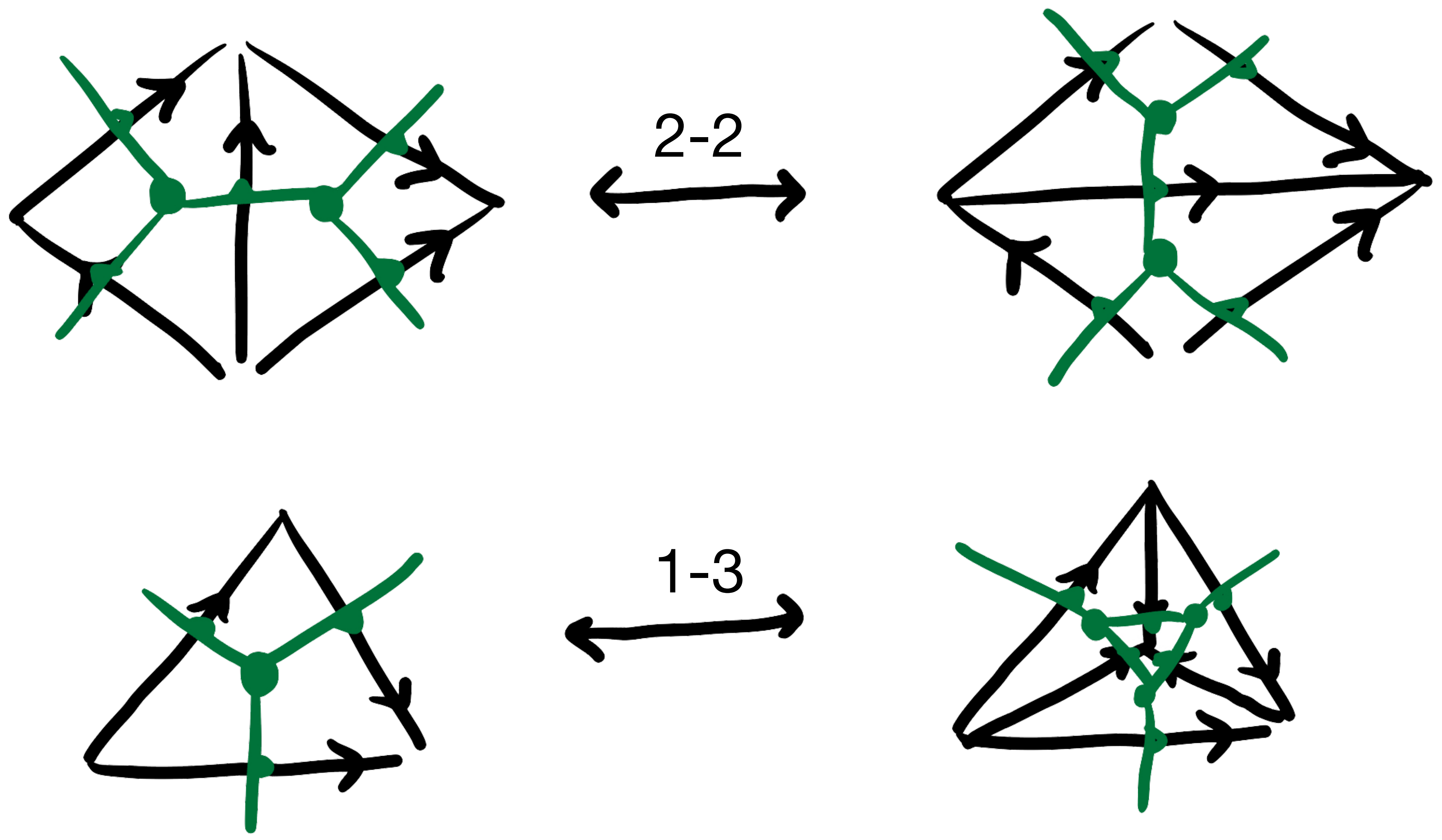
fixed-point models

bulk



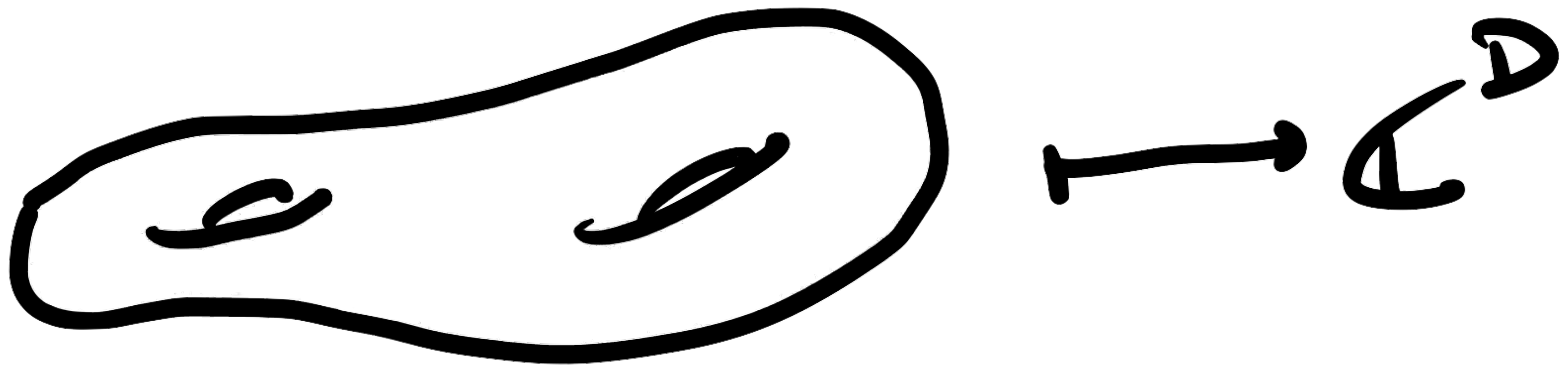
topological invariance by identifying states on **equivalent triangulations**

related by **Pachner moves**



fixed-point models

bulk



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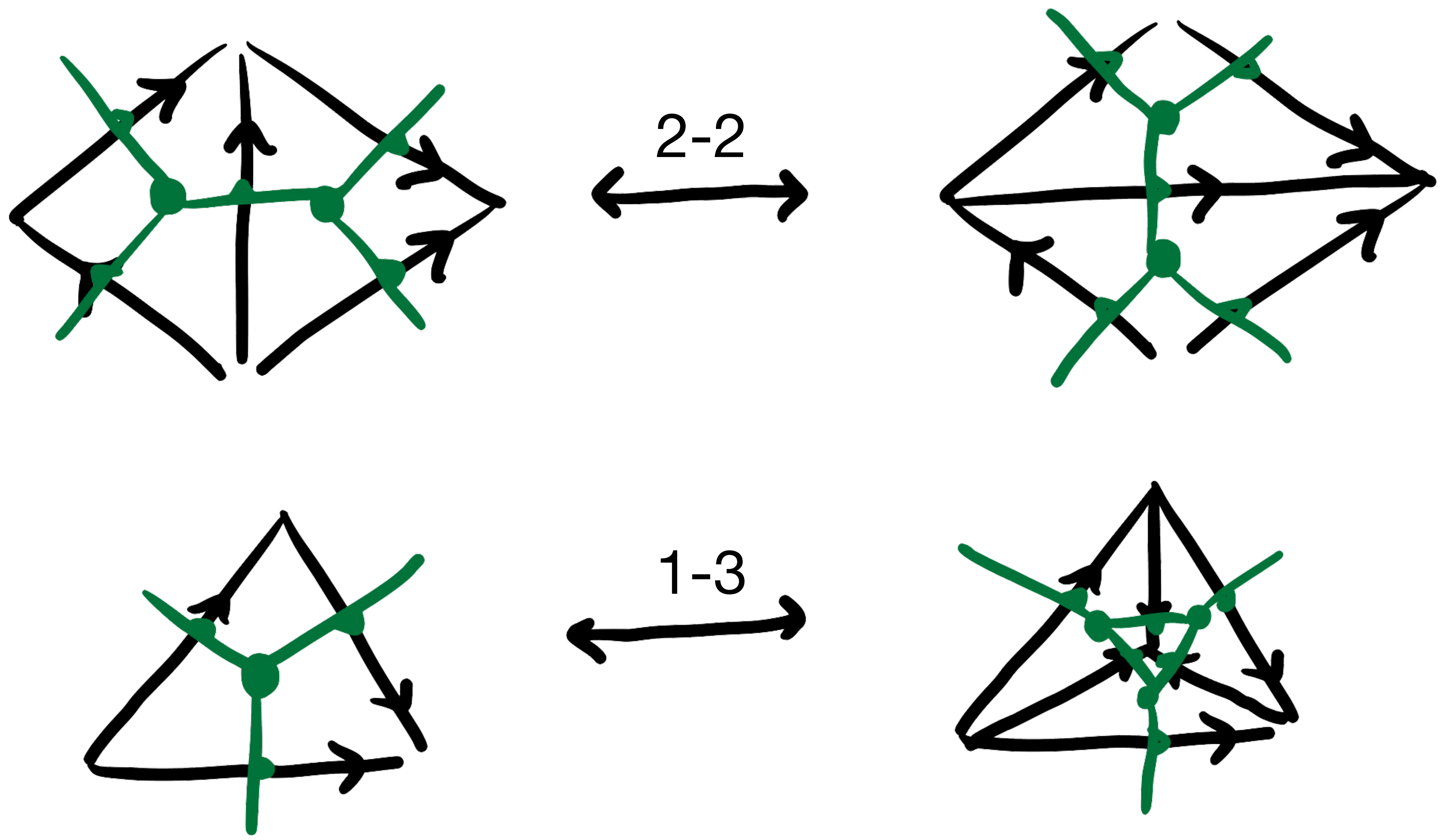
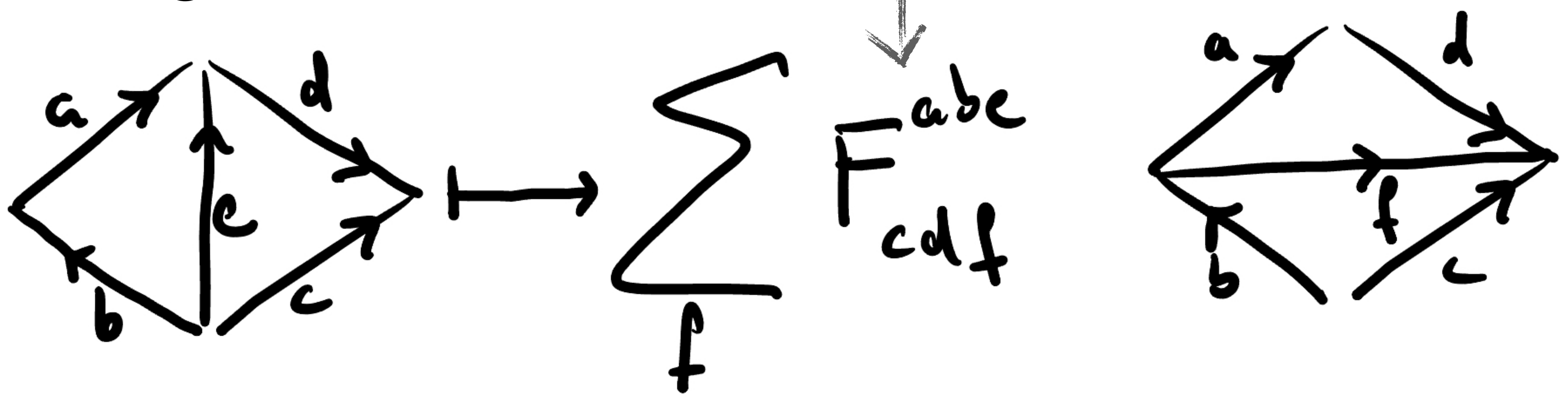


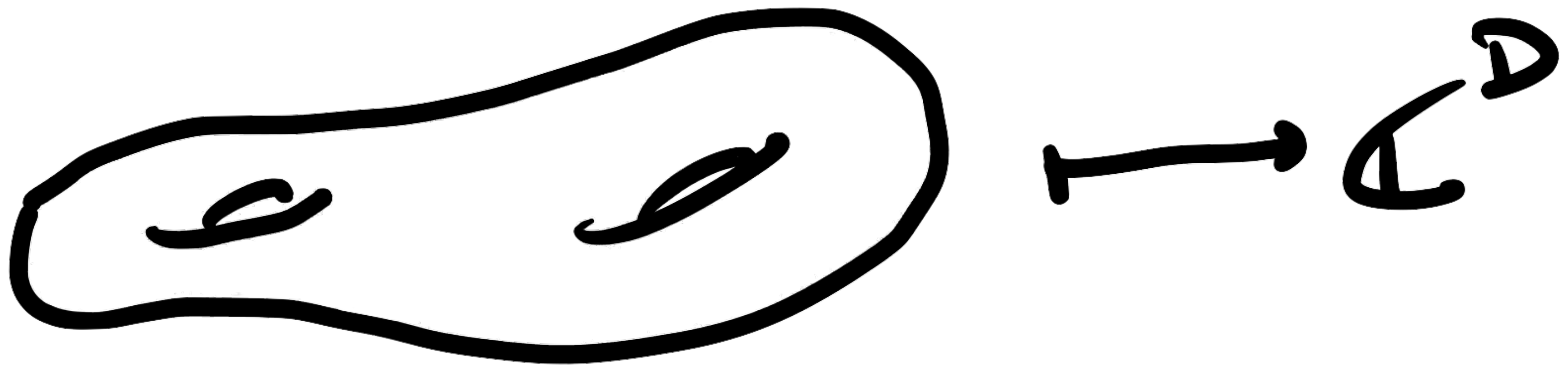
"F-symbol/tensor"

induce **linear maps** on associated spaces
e.g.



fixed-point models

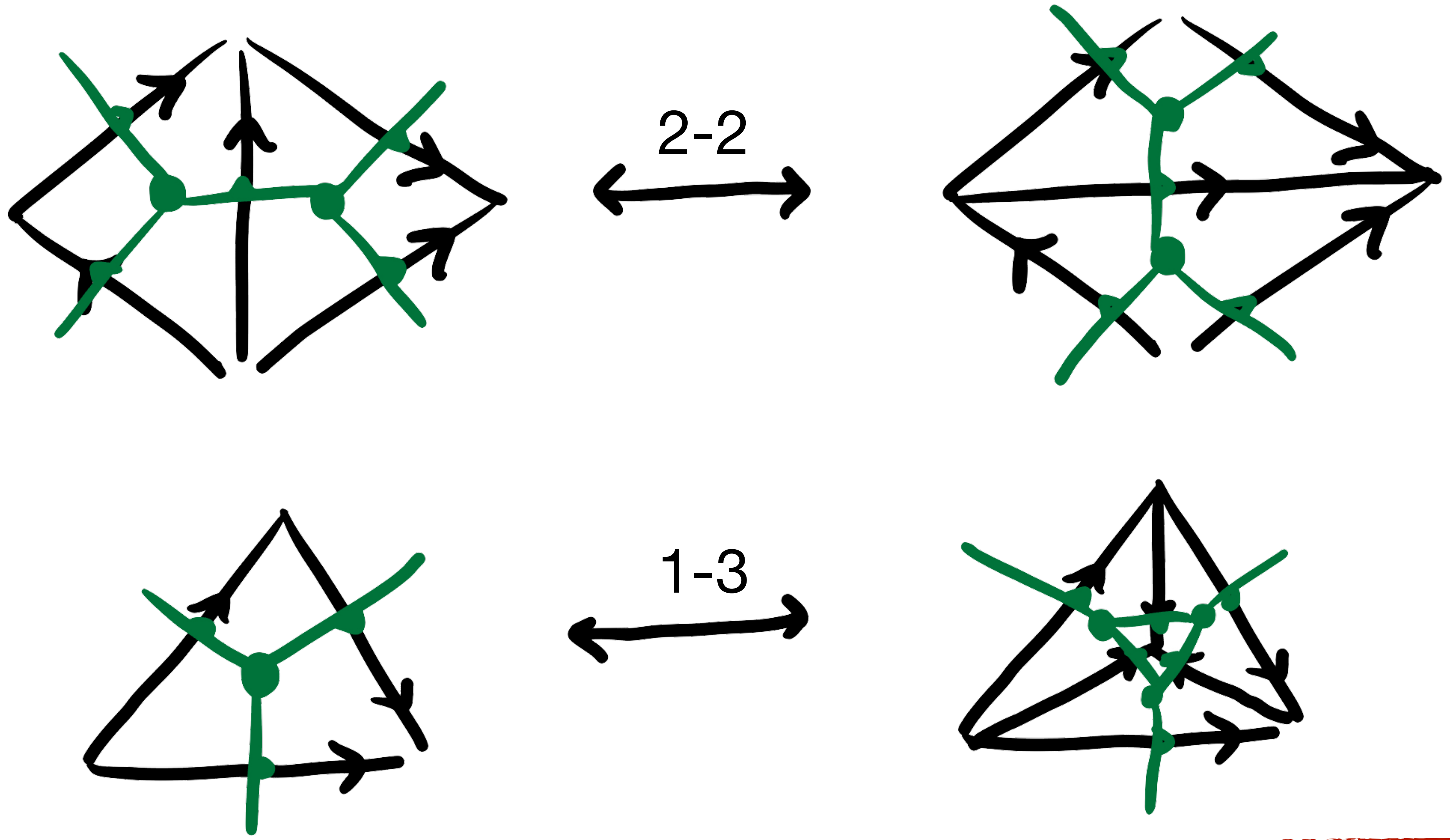
bulk



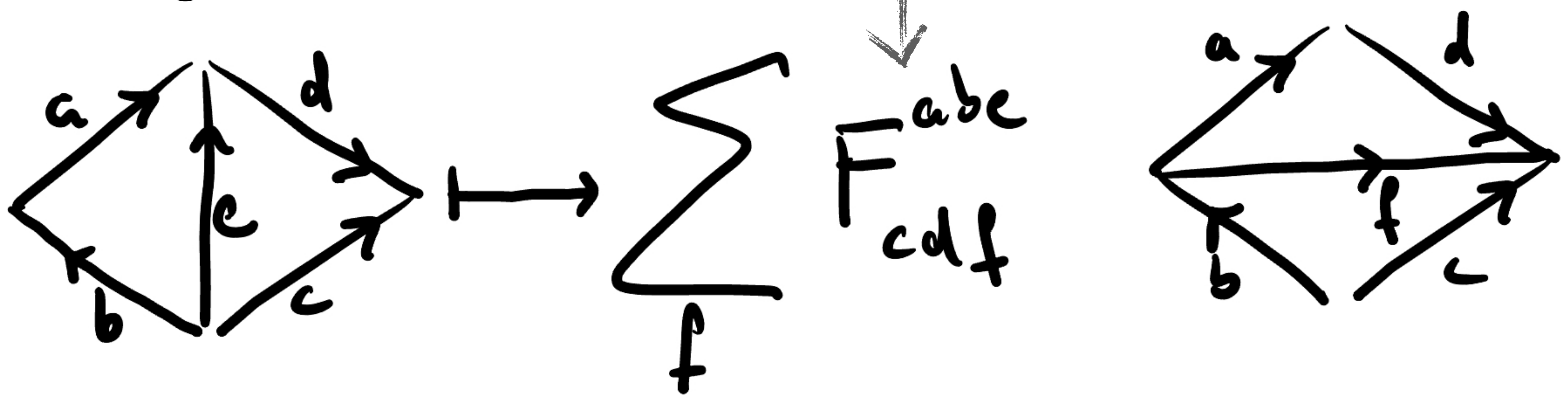
topological invariance by identifying states on **equivalent triangulations**

“F-symbol/tensor”

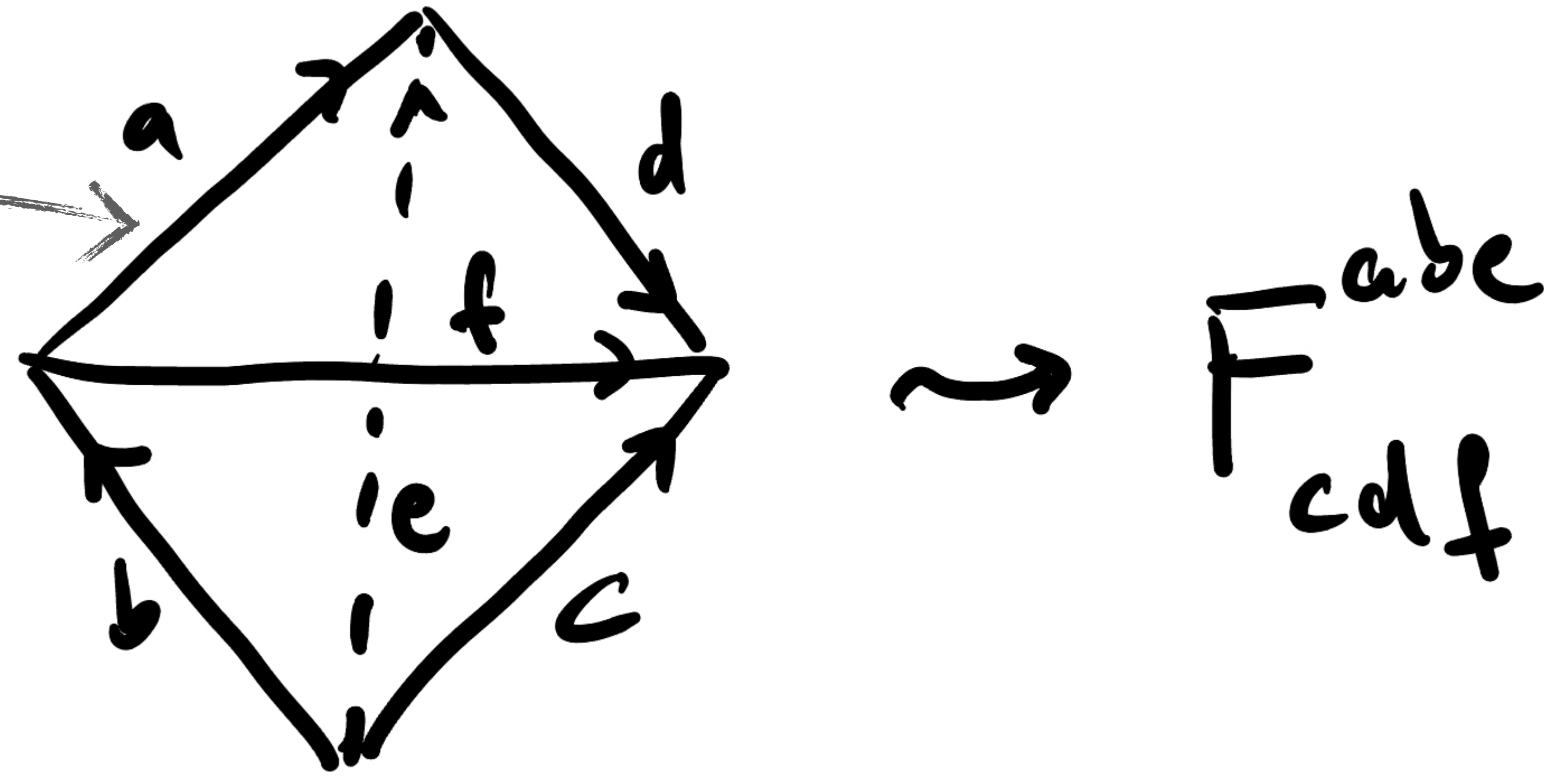
related by **Pachner moves**



induce **linear maps** on associated spaces
e.g.



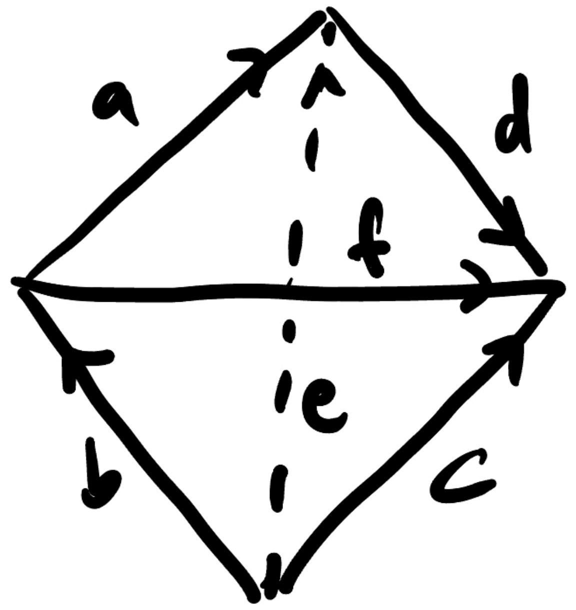
equivalent to assignment



building block of 3d triangulation

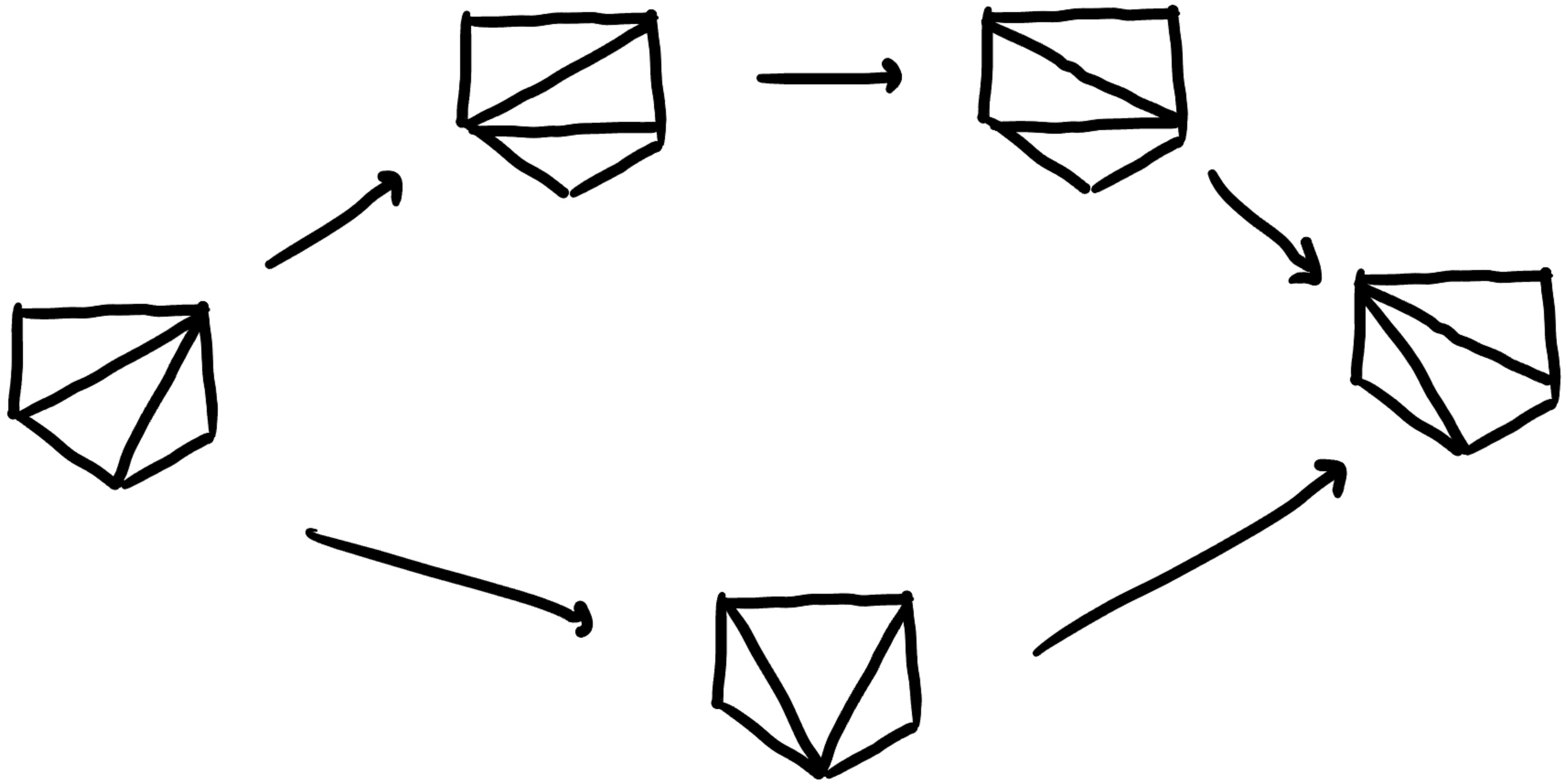
fixed-point models

bulk



$\rightsquigarrow F^{abe}_{cdf}$

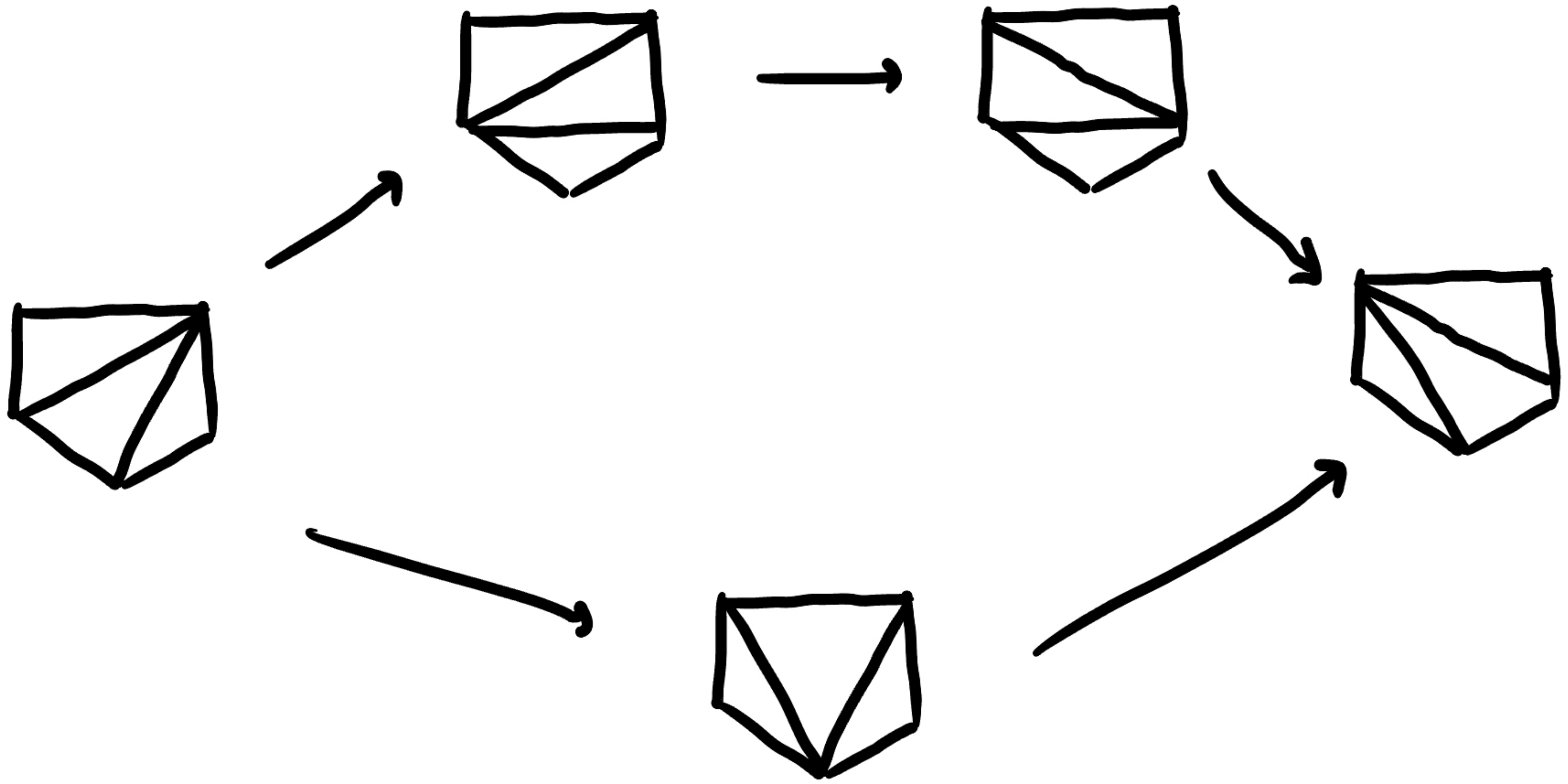
! sequence of Pachner moves is not unique !

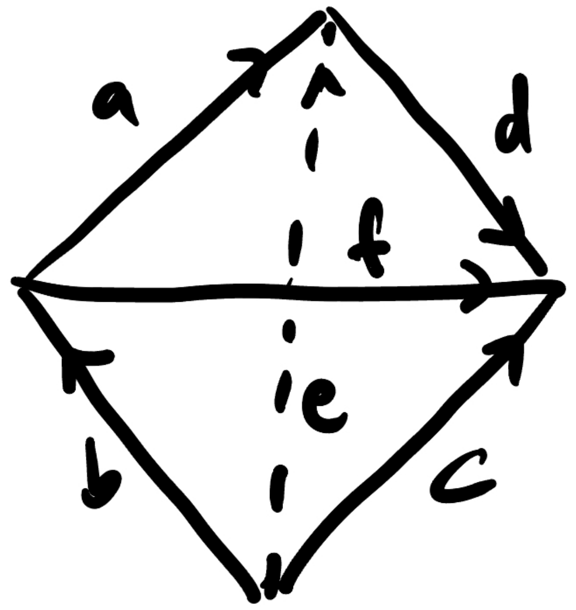


fixed-point models

bulk

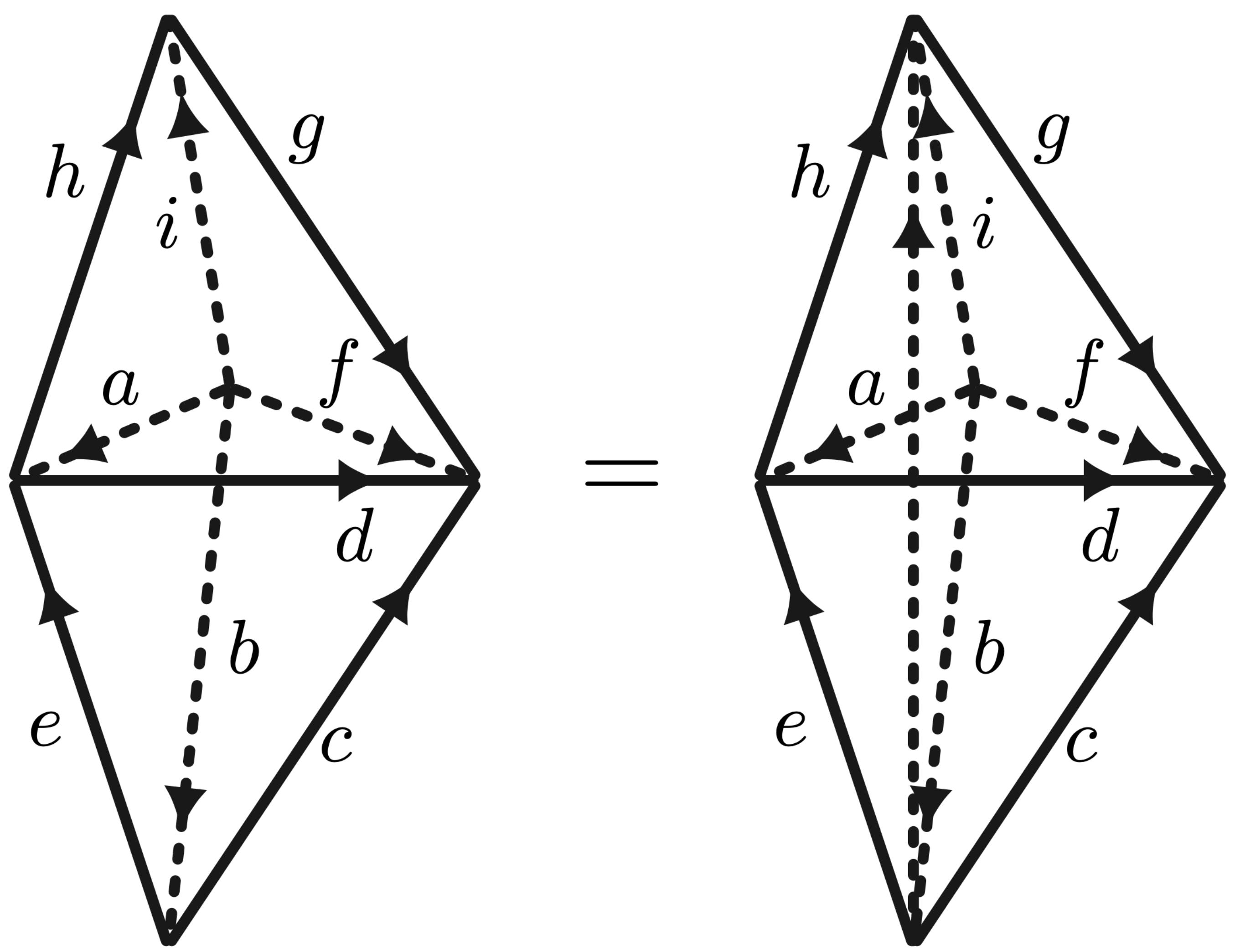
! sequence of Pachner moves is not unique !





$\rightsquigarrow F^{abe}_{cdf}$

F-symbols obey consistency condition



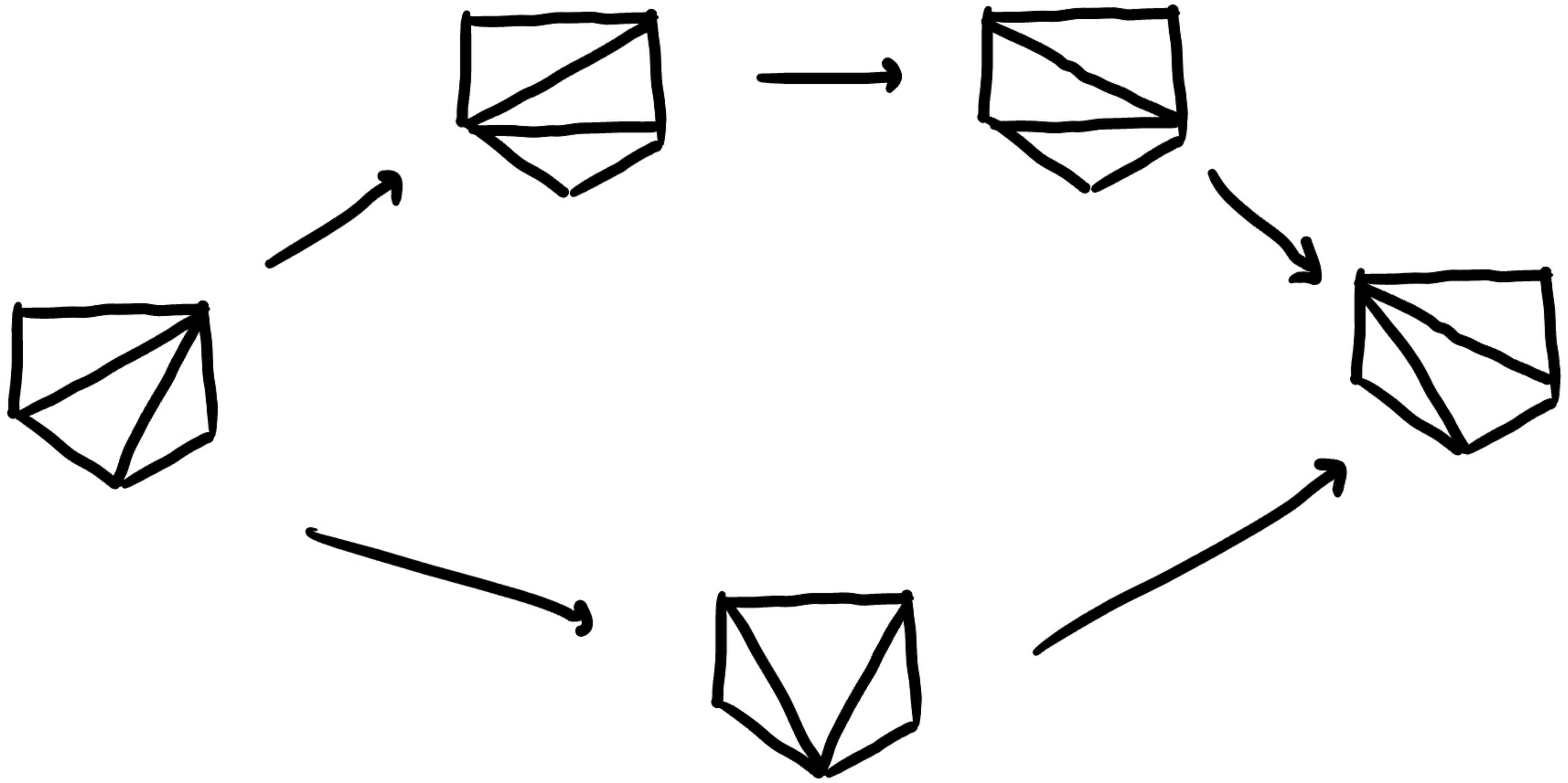
$$F^{eba}_{fdc} F^{hai}_{fgd} = \sum_k F^{hek}_{cgd} F^{kbi}_{fgc} F^{eba}_{ihk}$$

+ similar constraints for other branching structures

fixed-point models

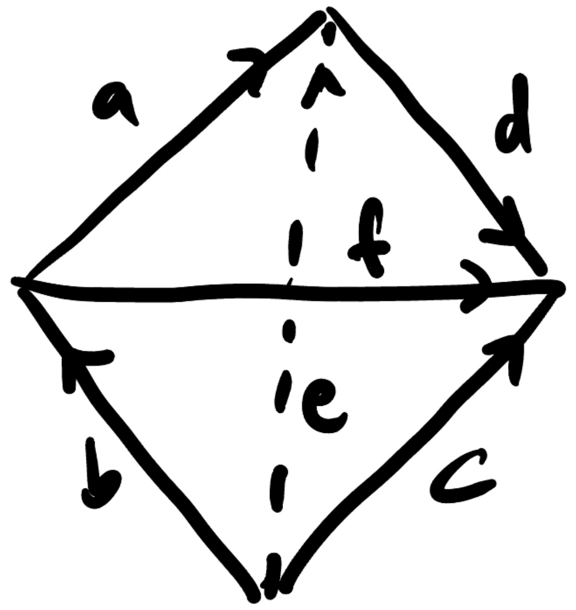
bulk

! sequence of Pachner moves is not unique !



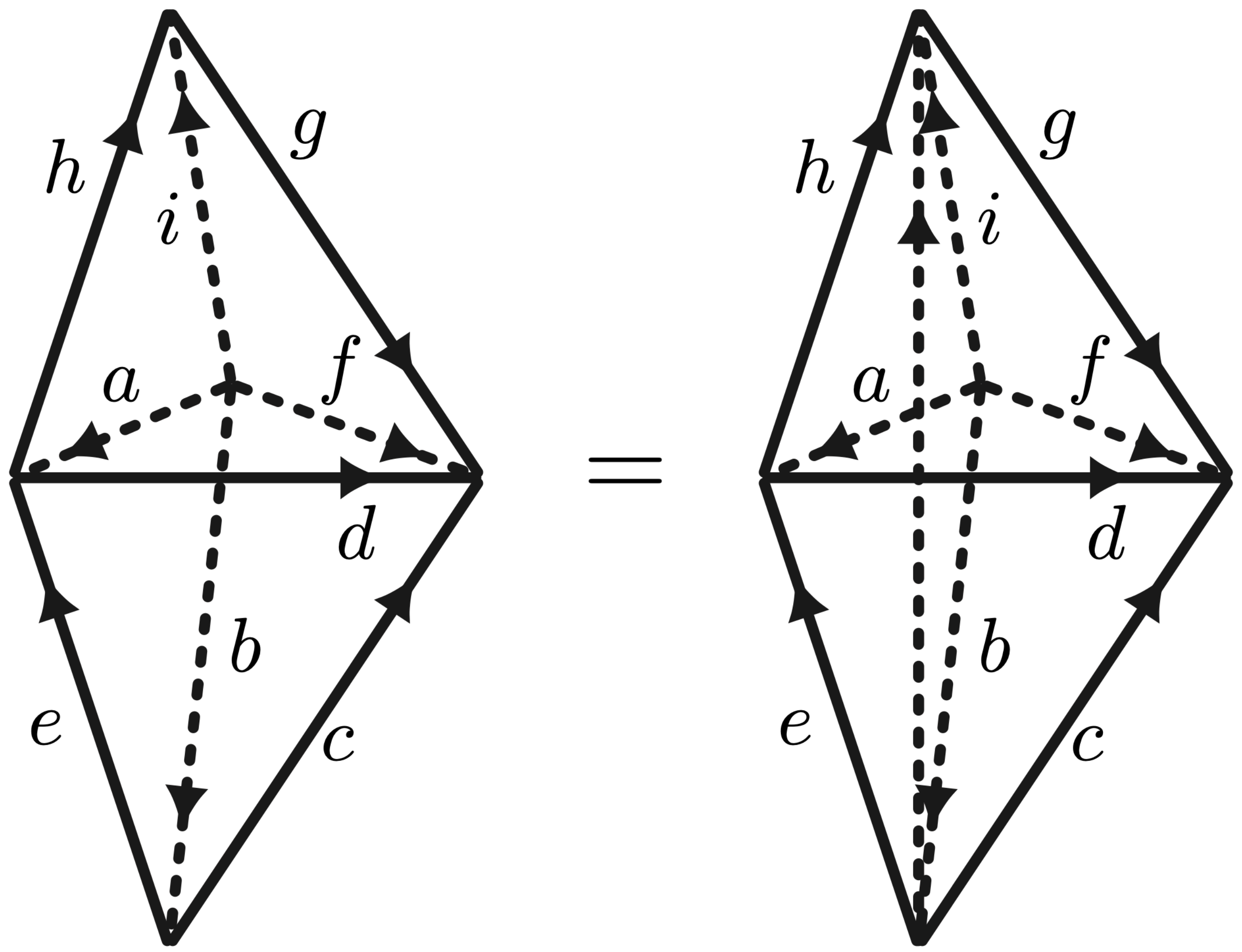
consistent data $\left\{ \{1, i, j, \dots\}, \{N_{ij}^k\}, \{F_{ijk}^{lmn}\} \right\}$

defines spherical fusion category



$\rightsquigarrow F^{abe}_{cdf}$

F-symbols obey consistency condition

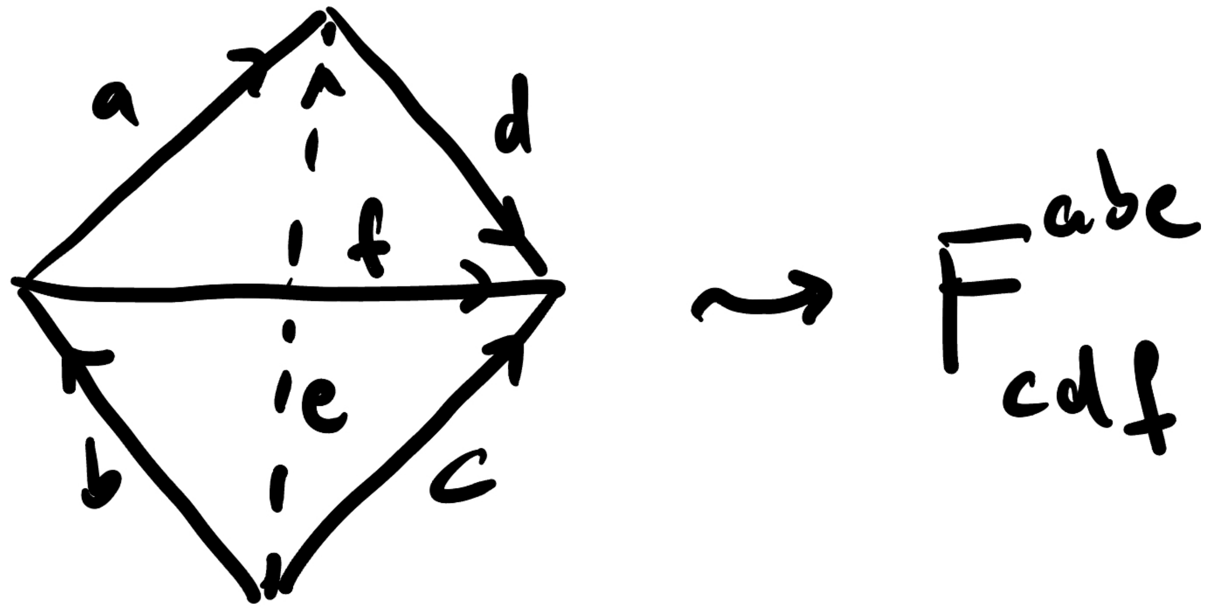


$$F^{eba}_{fdc}F^{hai}_{fgd} = \sum_k F^{hek}_{cgd}F^{kbi}_{fgc}F^{eba}_{ihk}$$

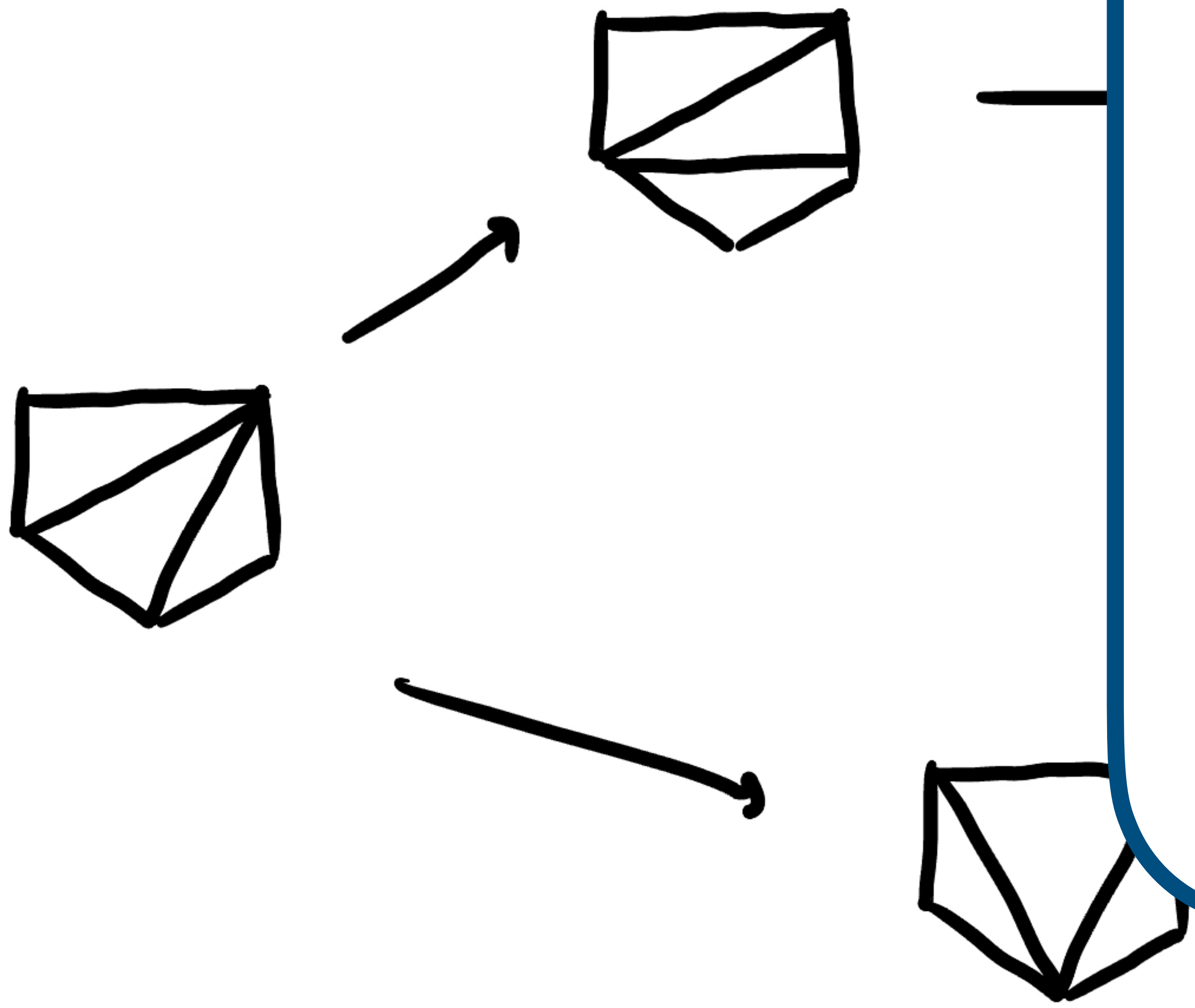
+ similar constraints for other branching structures

fixed-point models

bulk



! sequence of Pachner move



Example 1

finite group $G = \{1, i, j, k, \dots\}$

together with *3-cocycle* $\omega : G^3 \to \mathbb{C}^\times$ fulfilling

$$\omega(ab, c, d)\omega(a, b, cd) = \omega(b, c, d)\omega(a, bc, d)\omega(a, b, c)$$

defines spherical fusion category $\mathrm{Vec}^\omega(G)$

consistent data $\left\{ \{1, i, j, \dots\}, \{N_{ij}^k\}, \{F_{ijk}^{lmn}\} \right\}$

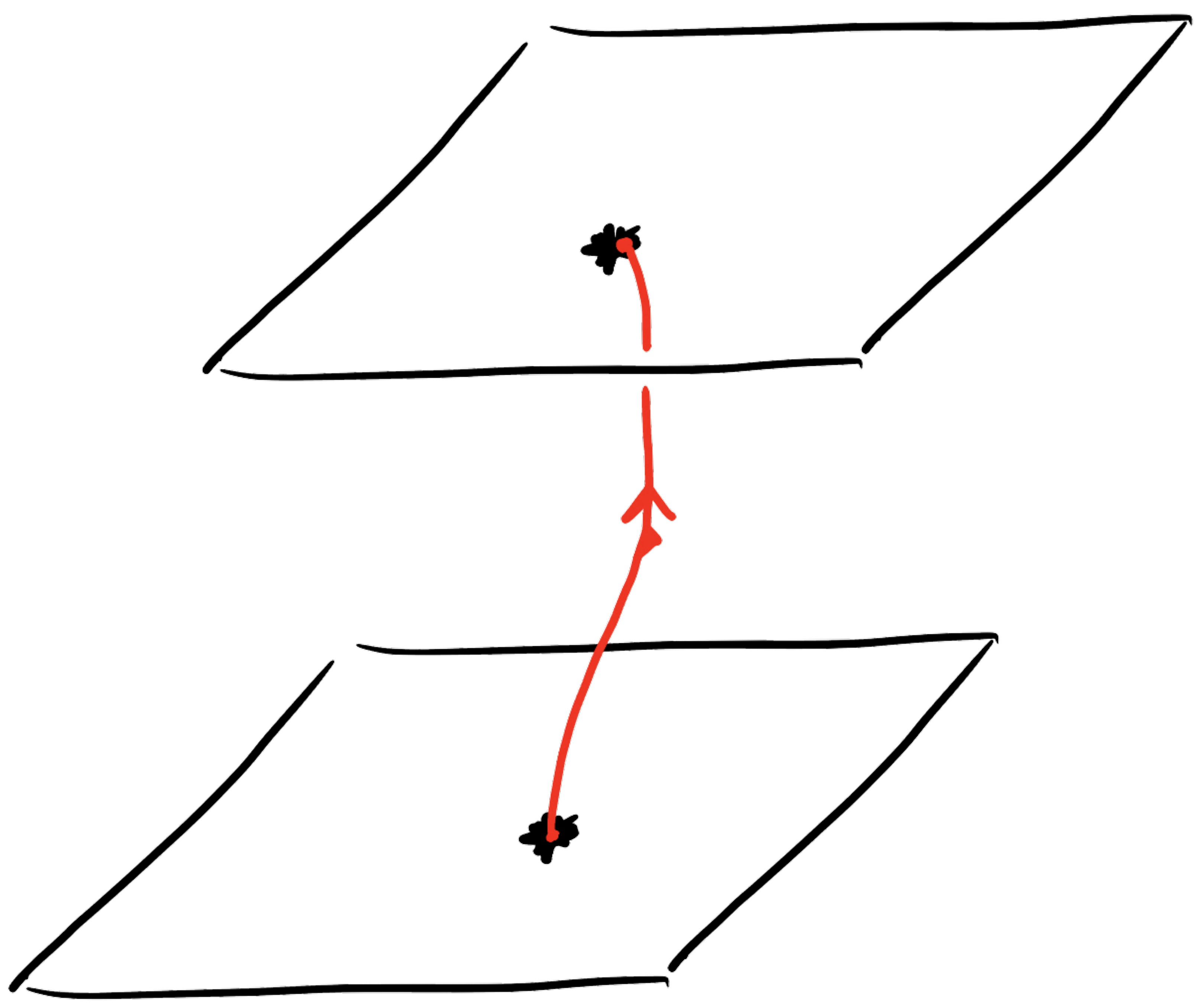
defines spherical fusion category

$$F_{fdc}^{eba} F_{fgd}^{hai} = \sum_k F_{cgd}^{hek} F_{fgc}^{kbi} F_{ihk}^{eba}$$

+ similar constraints for other branching structures

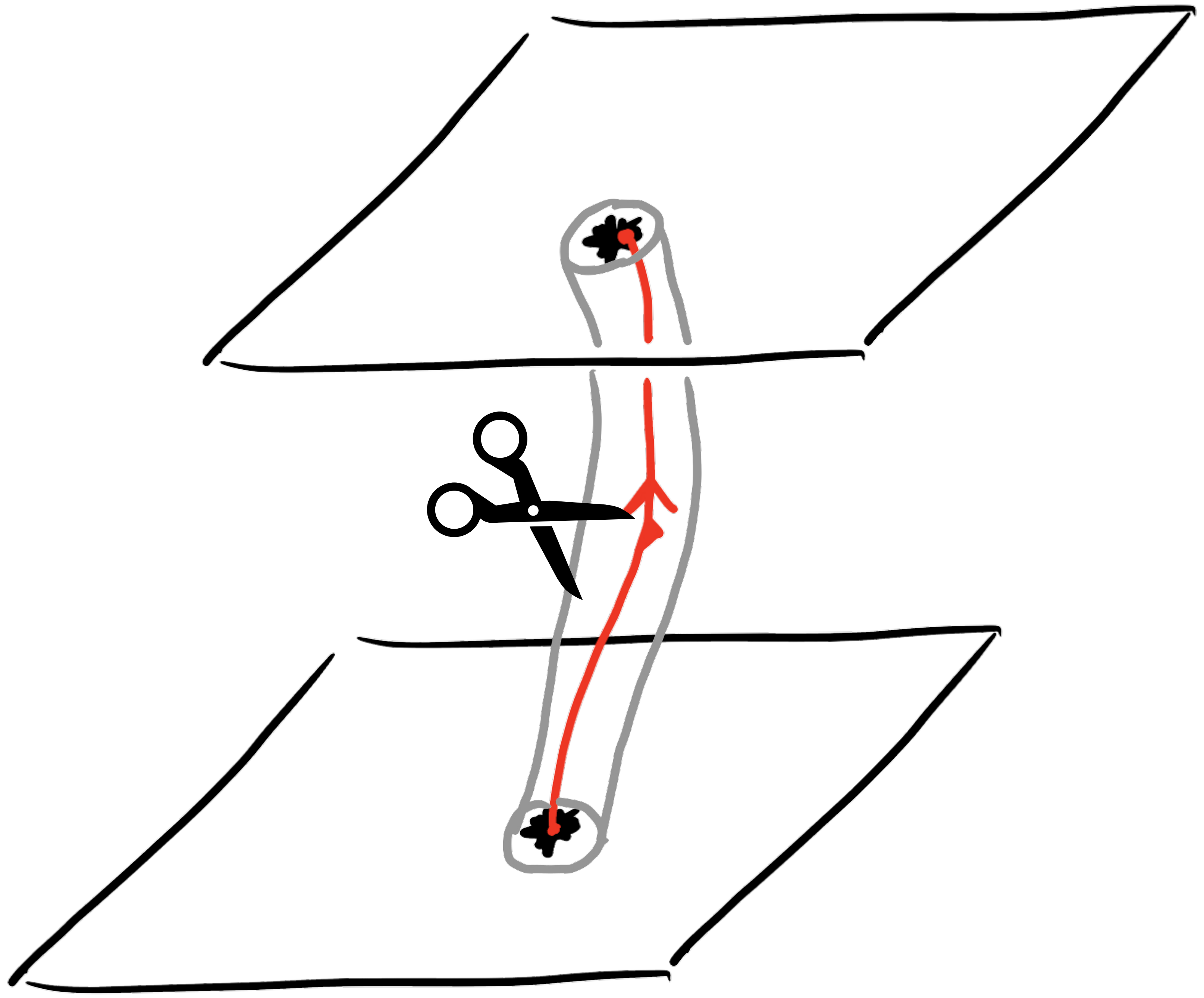
anyons in the bulk

tube algebra



anyons in the bulk

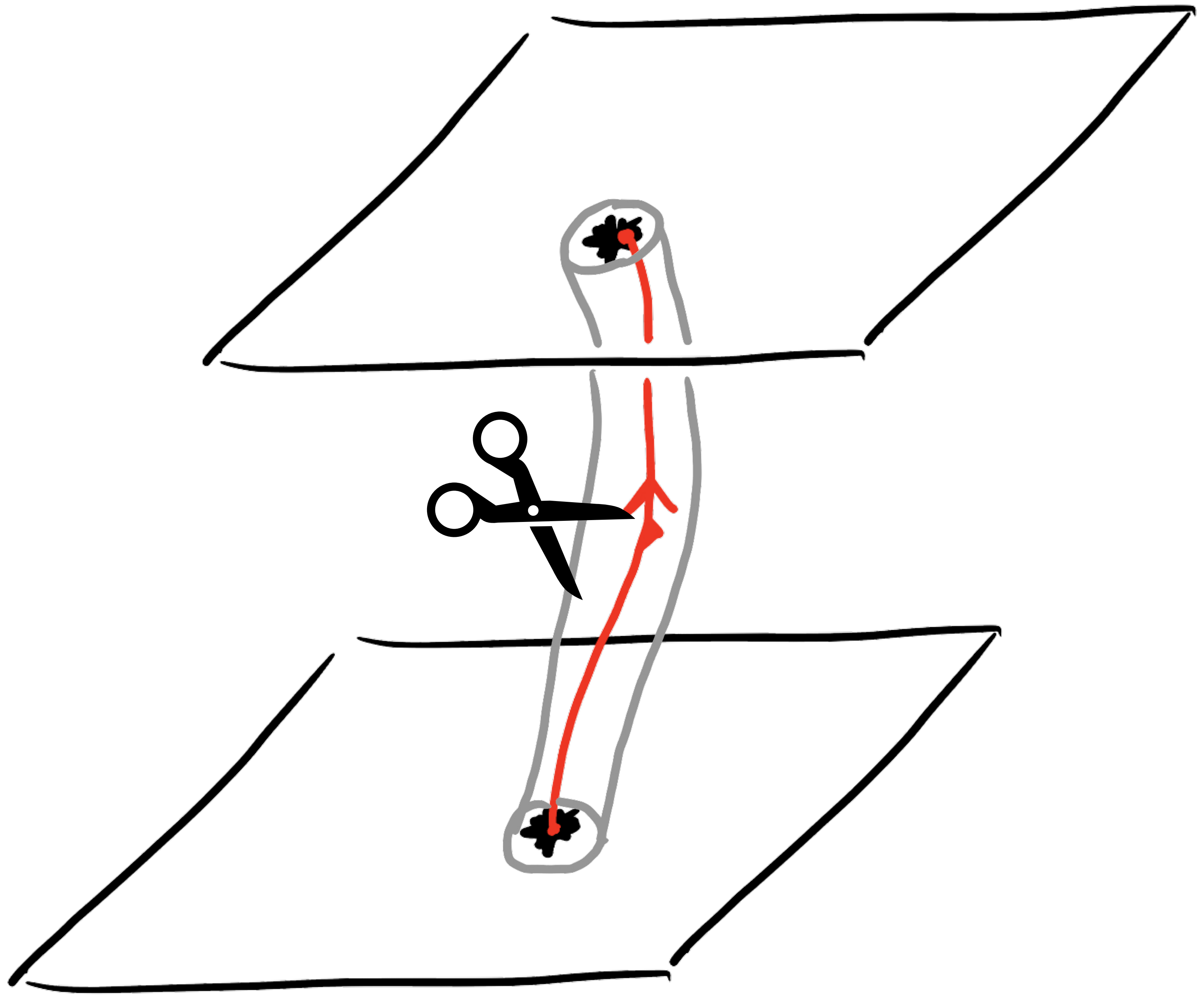
tube algebra

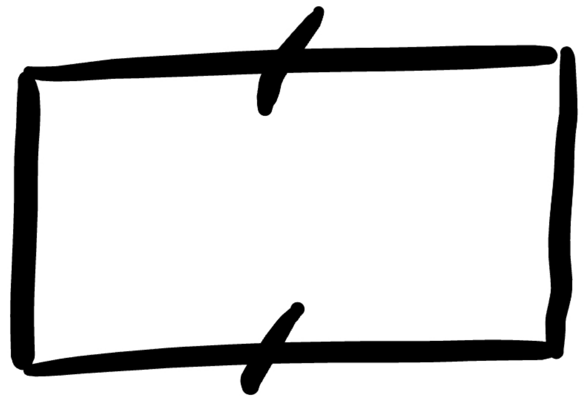


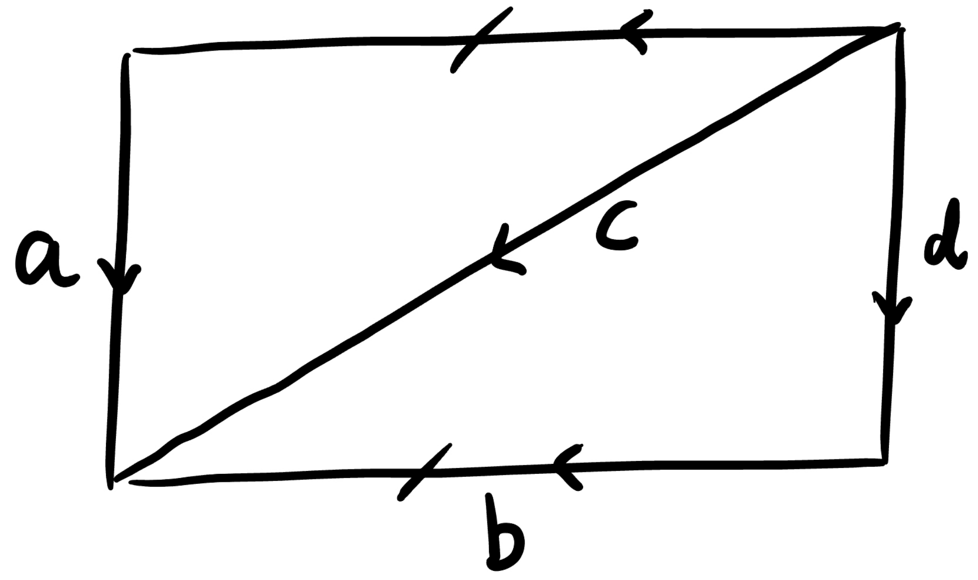
bulk line defect described by $S^1 \times [0,1] =: T$

anyons in the bulk

tube algebra



microscopic model:  $\mapsto V_T \simeq \mathbb{C}^{D_T}$

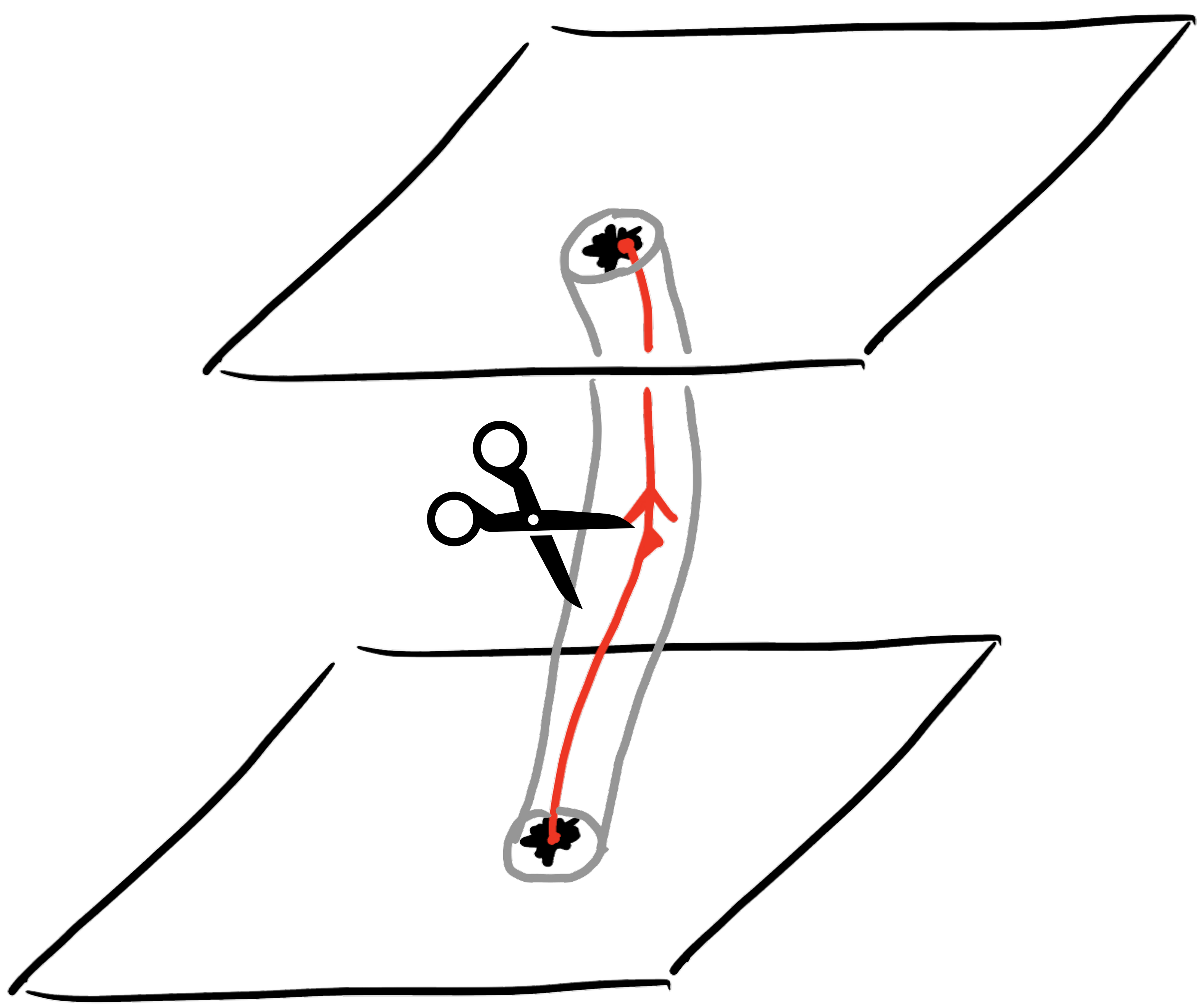
$V_T = \mathrm{span}_{\mathbb{C}} \Big\{$ 

$\Big\}$

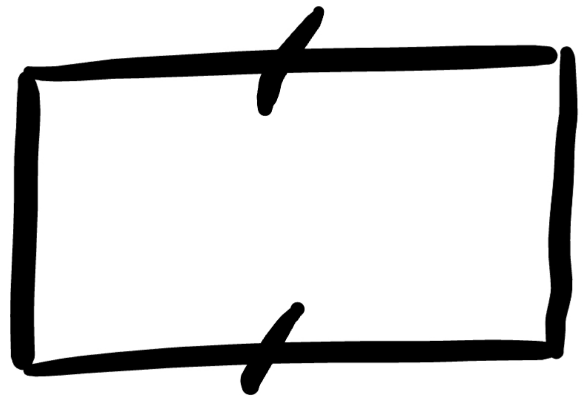
bulk line defect described by $S^1 \times [0,1] =: T$

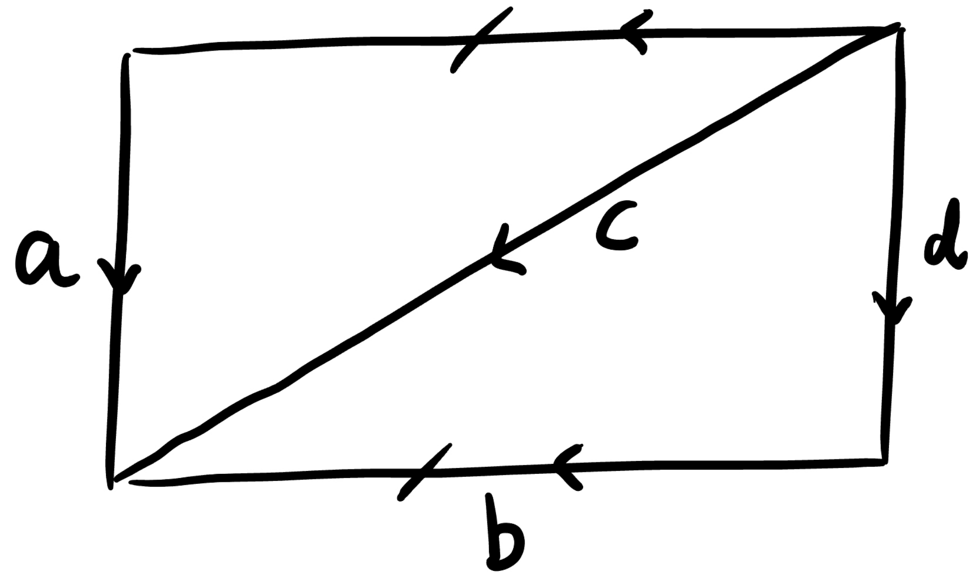
anyons in the bulk

tube algebra

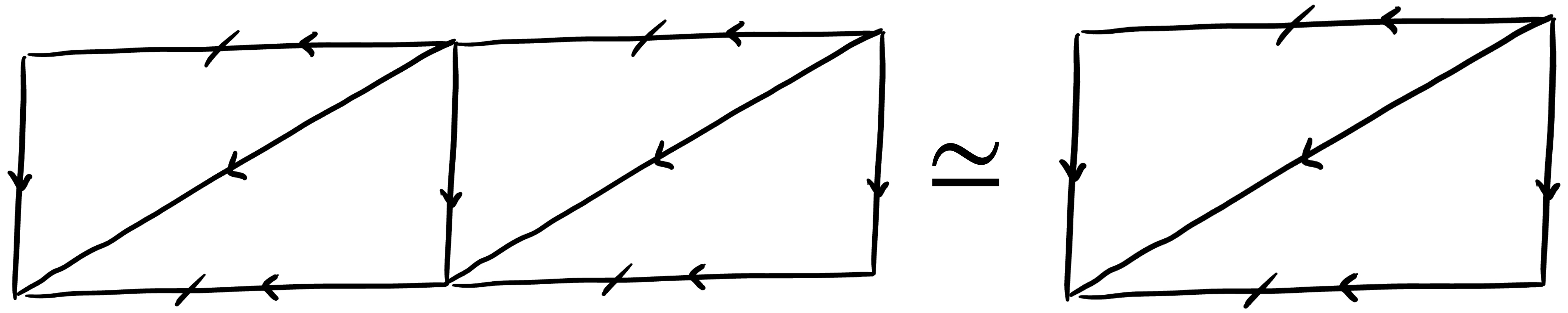


bulk line defect described by $S^1 \times [0,1] =: T$

microscopic model:  $\mapsto V_T \simeq \mathbb{C}^{D_T}$

$V_T = \mathrm{span}_{\mathbb{C}} \{$  $\}$

V_T is not only a vector space…

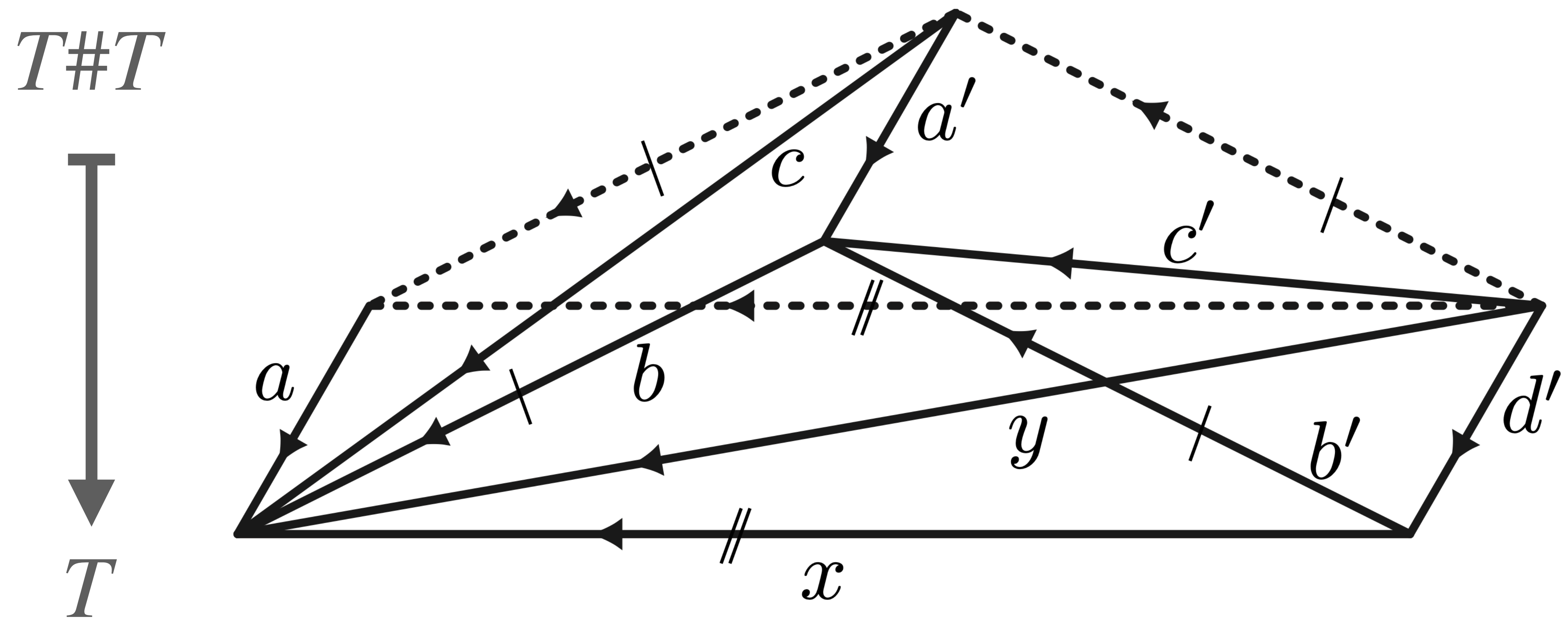


…but also an **algebra**

anyons in the bulk

tube algebra

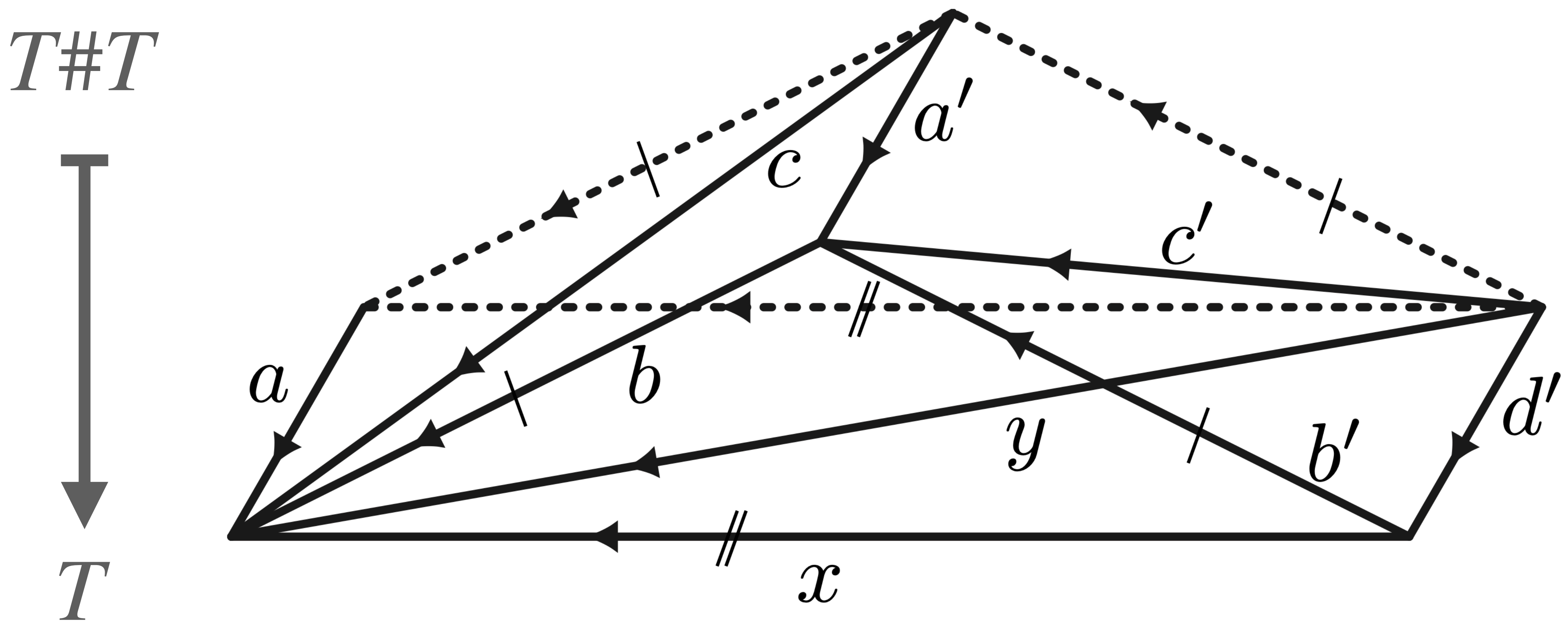
multiplication $*$ on V_T defined by **linear map** associated to

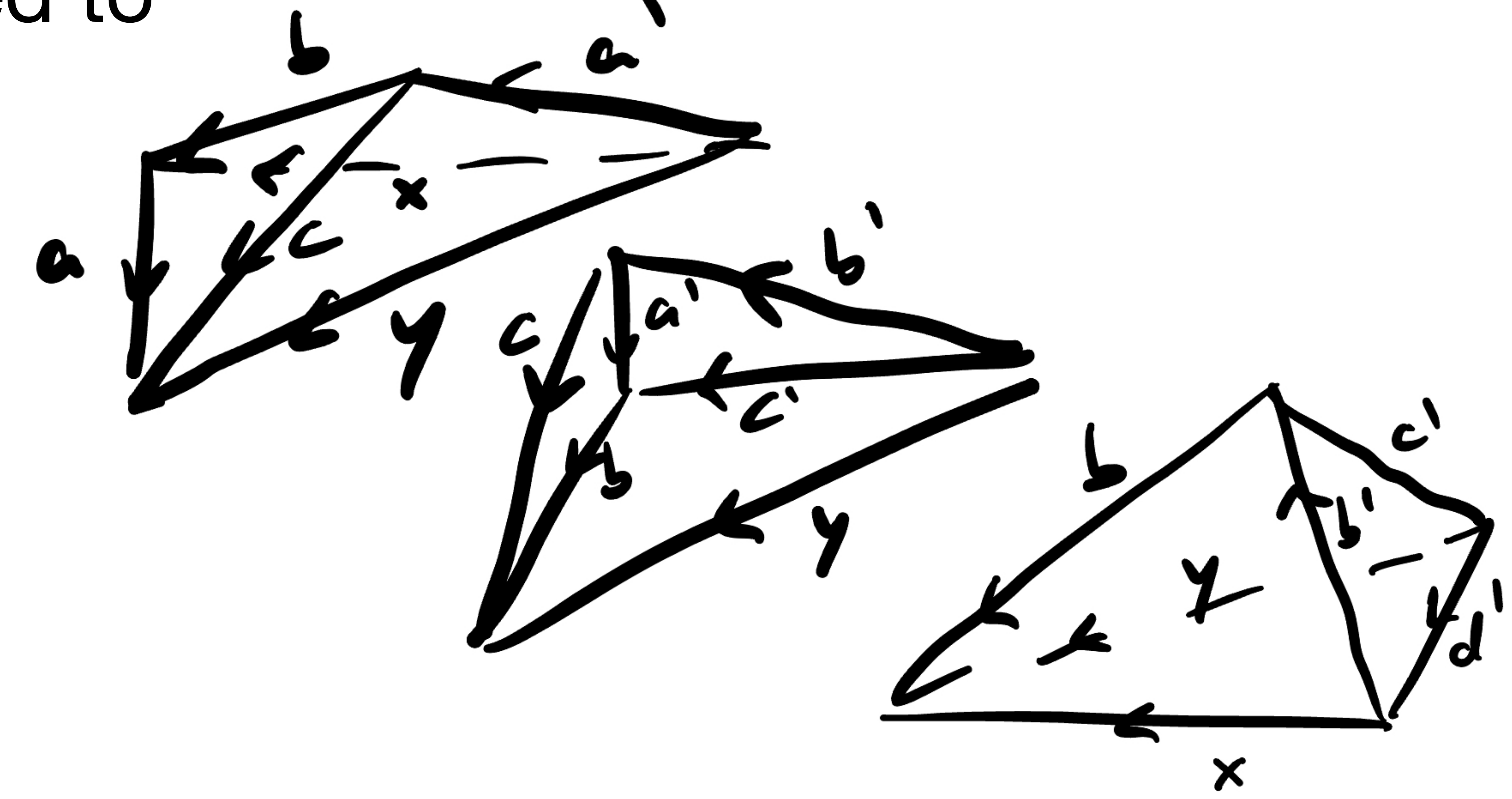


anyons in the bulk

tube algebra

multiplication $*$ on V_T defined by **linear map** associated to

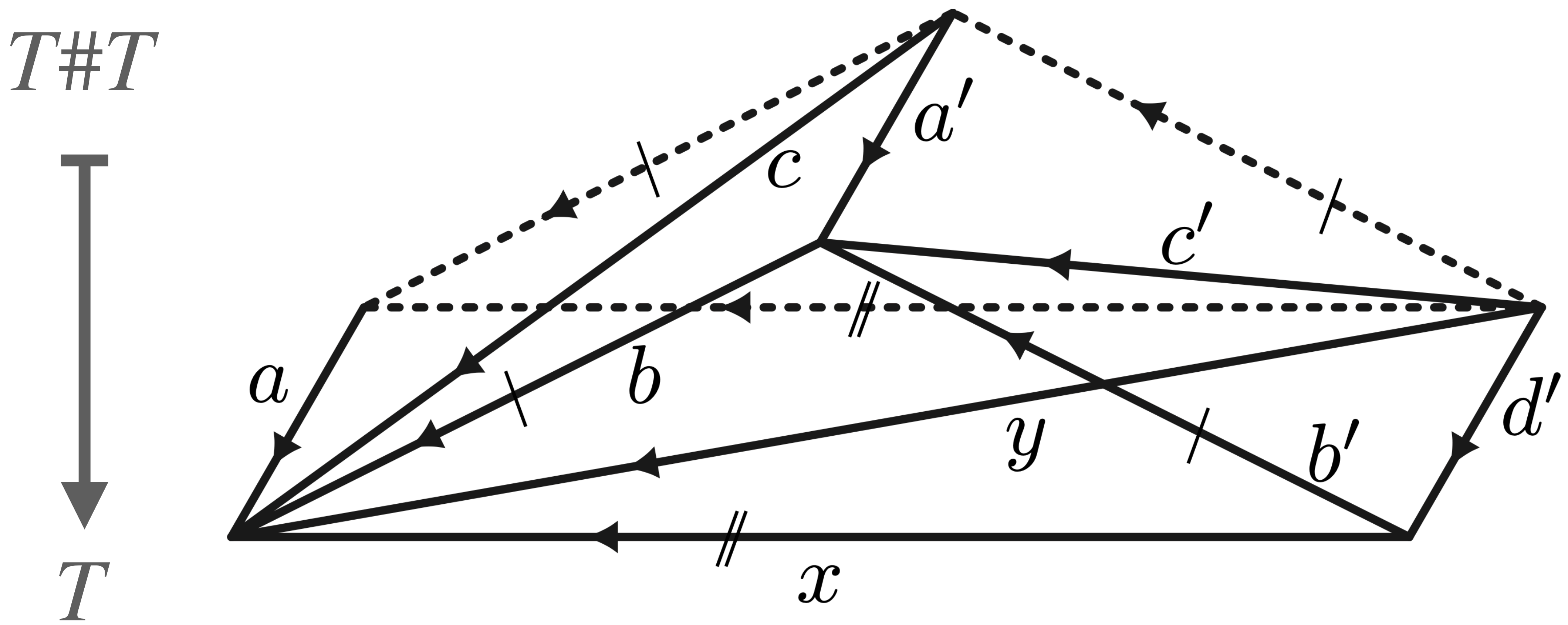


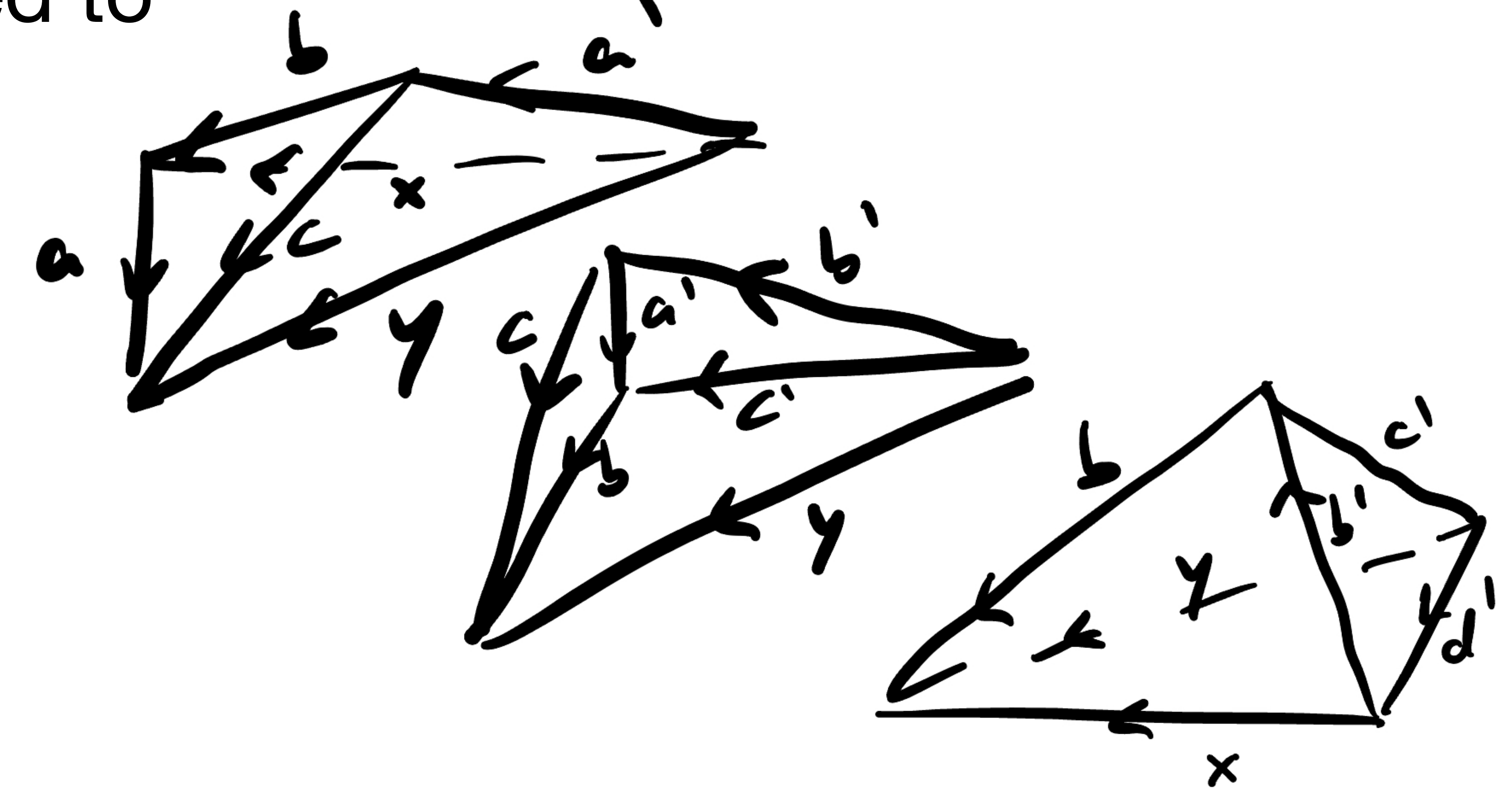


anyons in the bulk

tube algebra

multiplication $*$ on V_T defined by **linear map** associated to



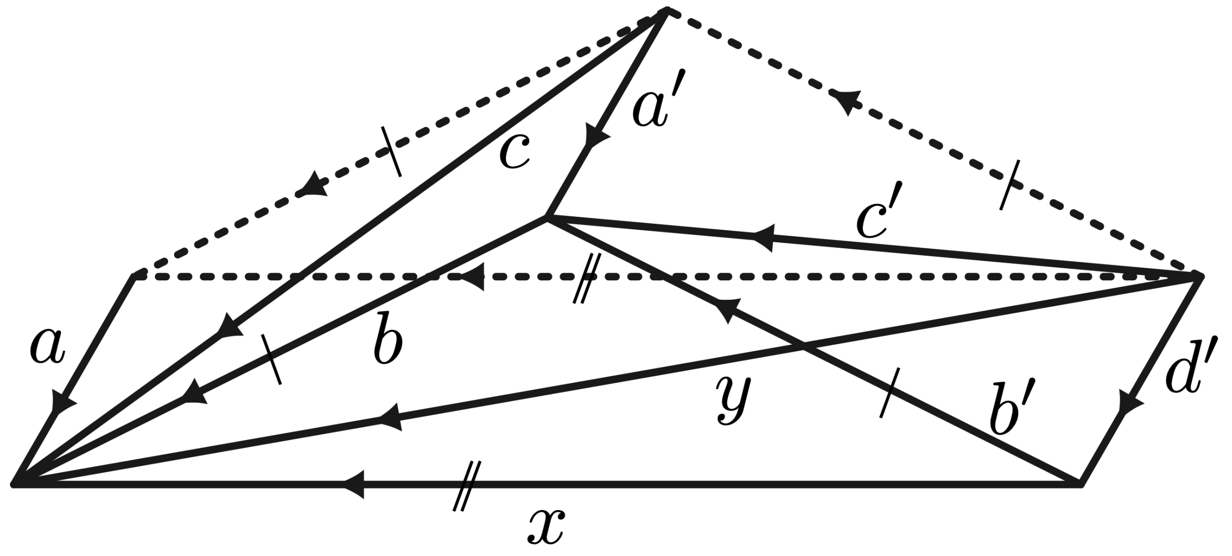


$$(a,b,c,d)_T * (a',b',c',d')_T = \delta_{d,a'} \sum_{x,y} F^{bb'x}_{yac} F^{b'd'c'}_{ybx} \overline{F^{a'b'c'}_{ybc}} (a,x,y,d')_T$$

as an algebra, $V_T = \bigoplus_i T_i$ with T_i irreducible subspaces

anyons in the bulk

tube algebra

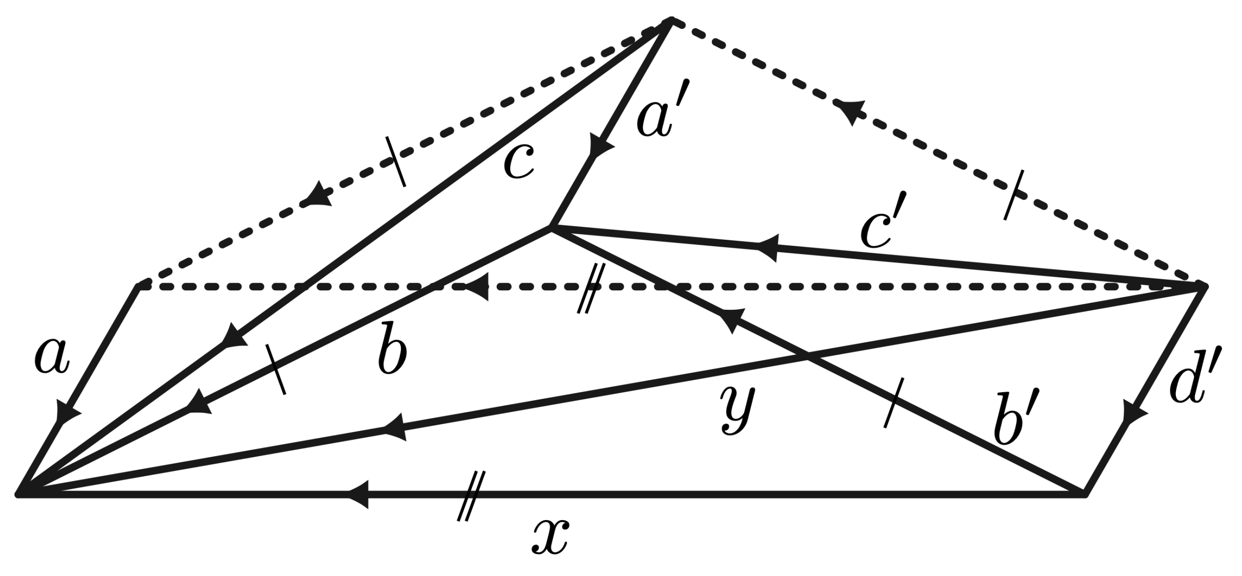


$\simeq FFF$

$V_T = \bigoplus_i T_i$ with T_i irreducible subspaces

anyons in the bulk

tube algebra



$\simeq FFF$

$V_T = \bigoplus_i T_i$ with T_i irreducible subspaces

center of V
$Z(V) = \{a \in V : a * v = v * a \ \forall v \in V\}$

$\{T_i$ irreducible$\}$ $\overset{\text{1-1}}{\longleftrightarrow}$ $\{c_i \in Z(V_T) : \ c_i * c_j = \delta_{i,j}\, c_i$ indecomposable$\}$

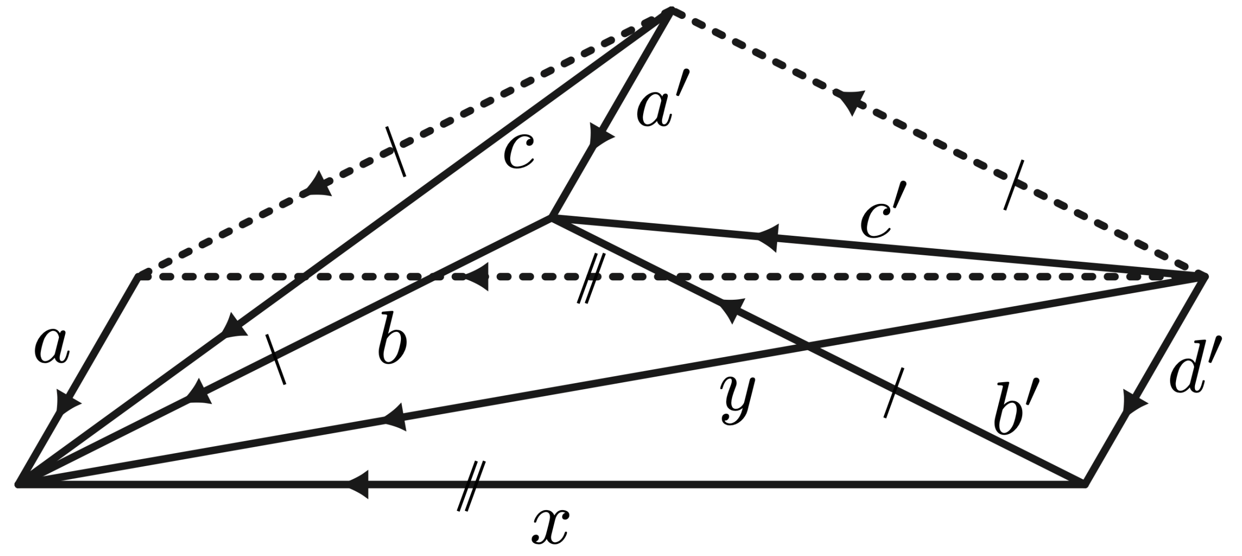
indecomposable central idempotents

decomposable:
$\exists$ central idempotents $a, b : c = a + b$

c_i can be viewed as **projector onto** T_i

anyons in the bulk

tube algebra



$\simeq FFF$

Example 1

Vec$^\omega(G)$ finite group G and (normalized) 3-cocycle ω

$$(g', h')_T * (g, h)_T = \delta_{g', hgh^{-1}} \beta_g(h', h)(g, h'h)_T$$

with $\beta_g(h', h) = \omega(h'hg(h'h)^{-1}, h', h)\, \omega(h', h, g)\, \overline{\omega(h', hgh^{-1}, h)}$

irreps 1-1 with (c, ρ_c) and associated central idempotents are

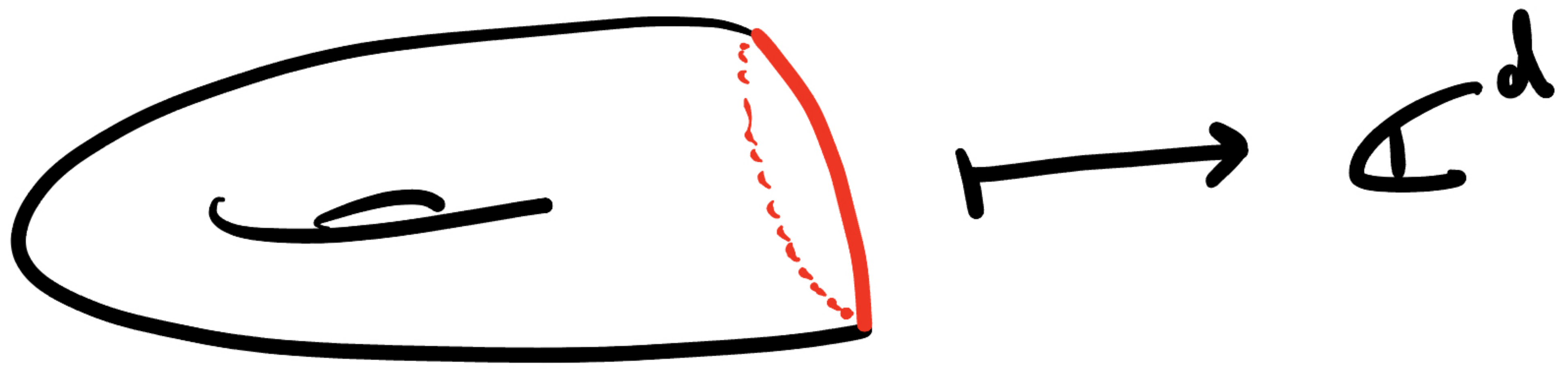
$$c^T_{(c,\rho_c)} = \frac{\dim(\rho_c)}{|Z(c)|} \sum_{g\in c} \sum_{h \in Z(g)} \overline{\tilde{\chi}^g_{\rho_c}(h)}(g, h)_T$$

conjugacy class: c

β_g-**projective** *Irrep of centralizer* $Z(c)$: ρ_c

fixed-point models

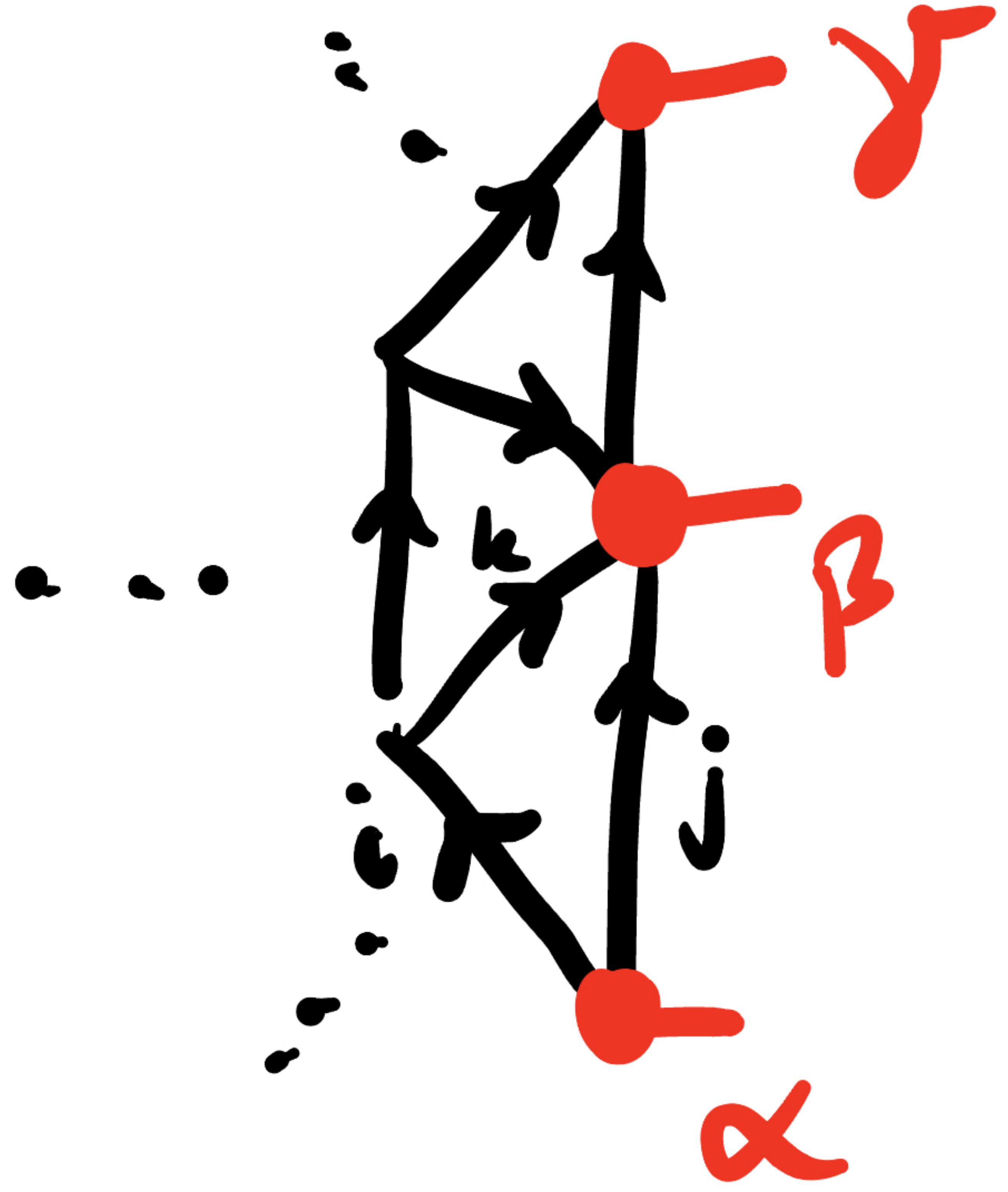
boundary



modify triangulation@ boundary: additional **degrees of freedom on boundary vertices**

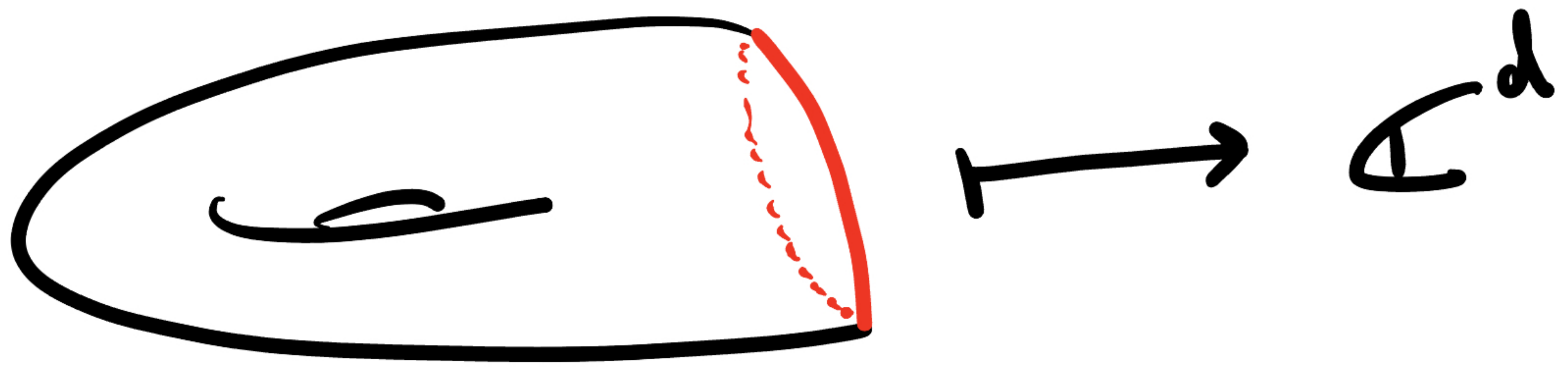
states embedded into **finite-dim. tensor product space**

$$\mathscr{H} = \bigotimes_{\text{edges}} \text{span}\{1, i, j, k, \dots\} \bigotimes_{\text{bdr'y vertices}} \text{span}\{\alpha, \beta, \dots\}$$



fixed-point models

boundary



modify triangulation@ boundary: additional **degrees of freedom on boundary vertices**

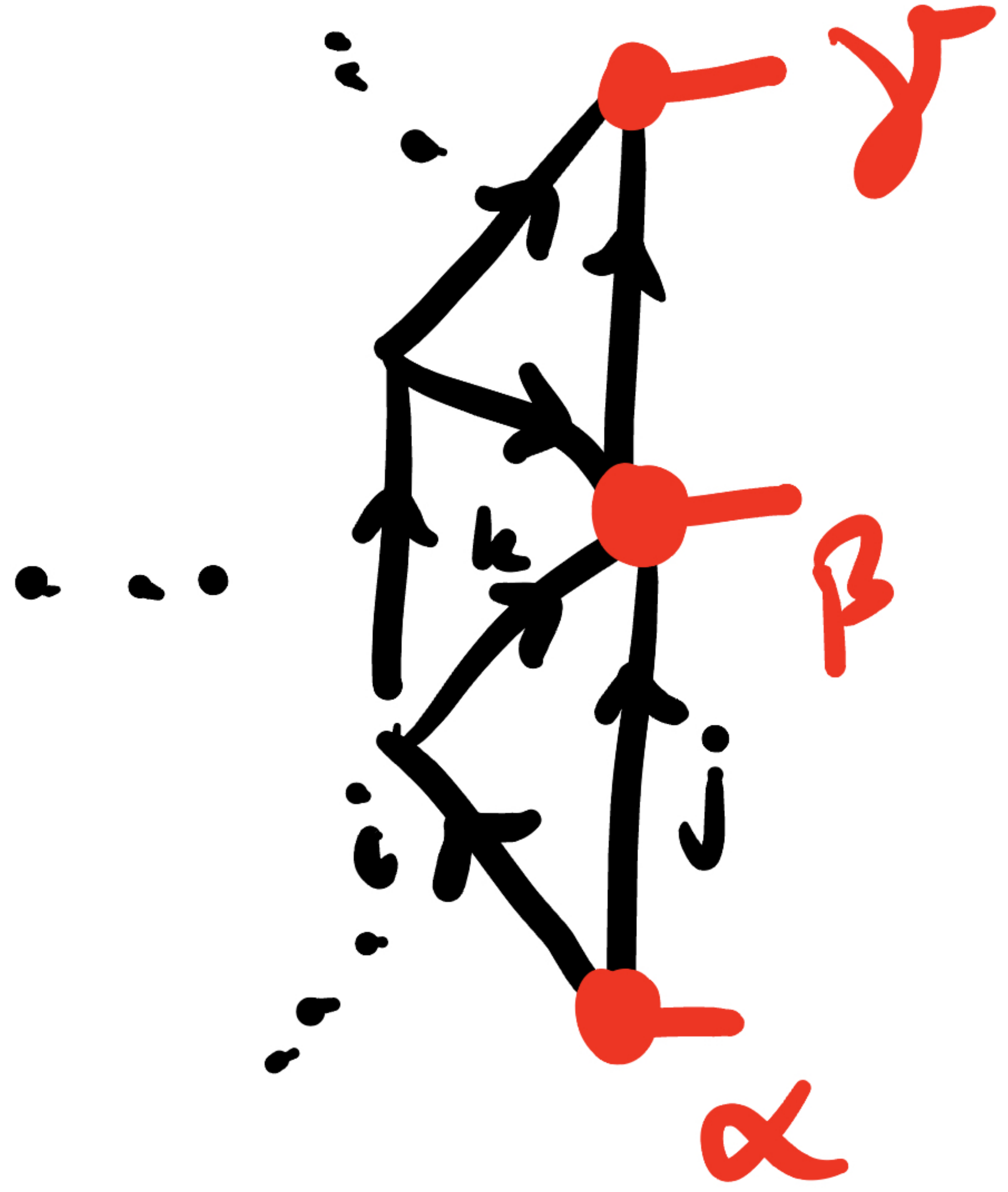
states embedded into **finite-dim. tensor product space**

$$\mathscr{H} = \bigotimes_{\text{edges}} \text{span}\{1, i, j, k, \ldots\} \bigotimes_{\text{bdr'y vertices}} \text{span}\{\alpha, \beta, \ldots\}$$

with **additional local constraints** @ each boundary edge

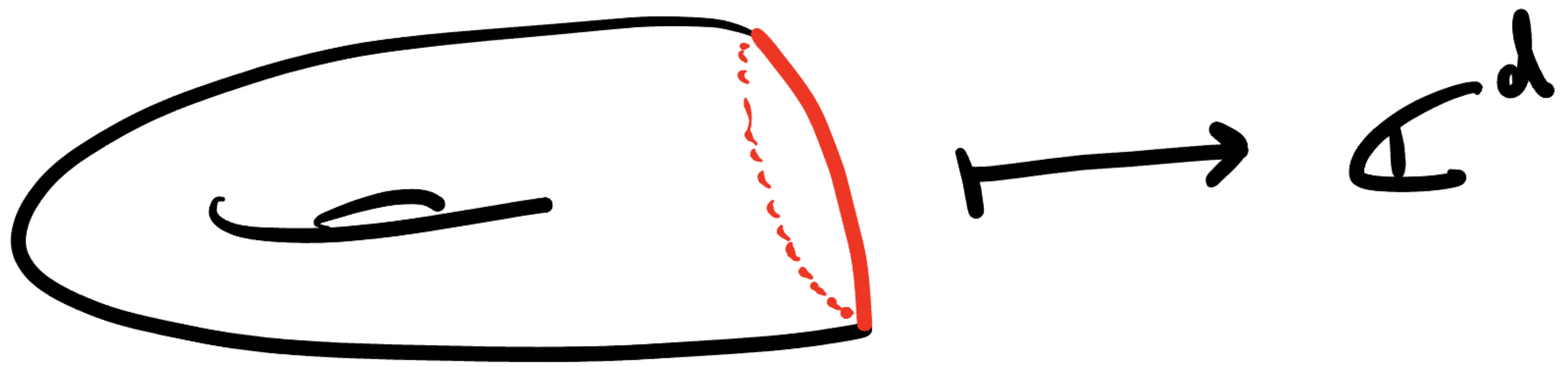
$$\boxed{i \triangleright \alpha = \bigoplus_{\beta} M_{i,\alpha}^{\beta}\, \beta, \quad M_{i,\alpha}^{\beta} \in \mathbb{Z}^+}$$

consistent with bulk N_{ij}^k $\quad i \triangleright (j \triangleright \alpha) \simeq (i \times j) \triangleright \alpha, \quad \forall i, j, \alpha$



fixed-point models

boundary



modify triangulation@ boundary: additional **degrees of freedom on boundary vertices**

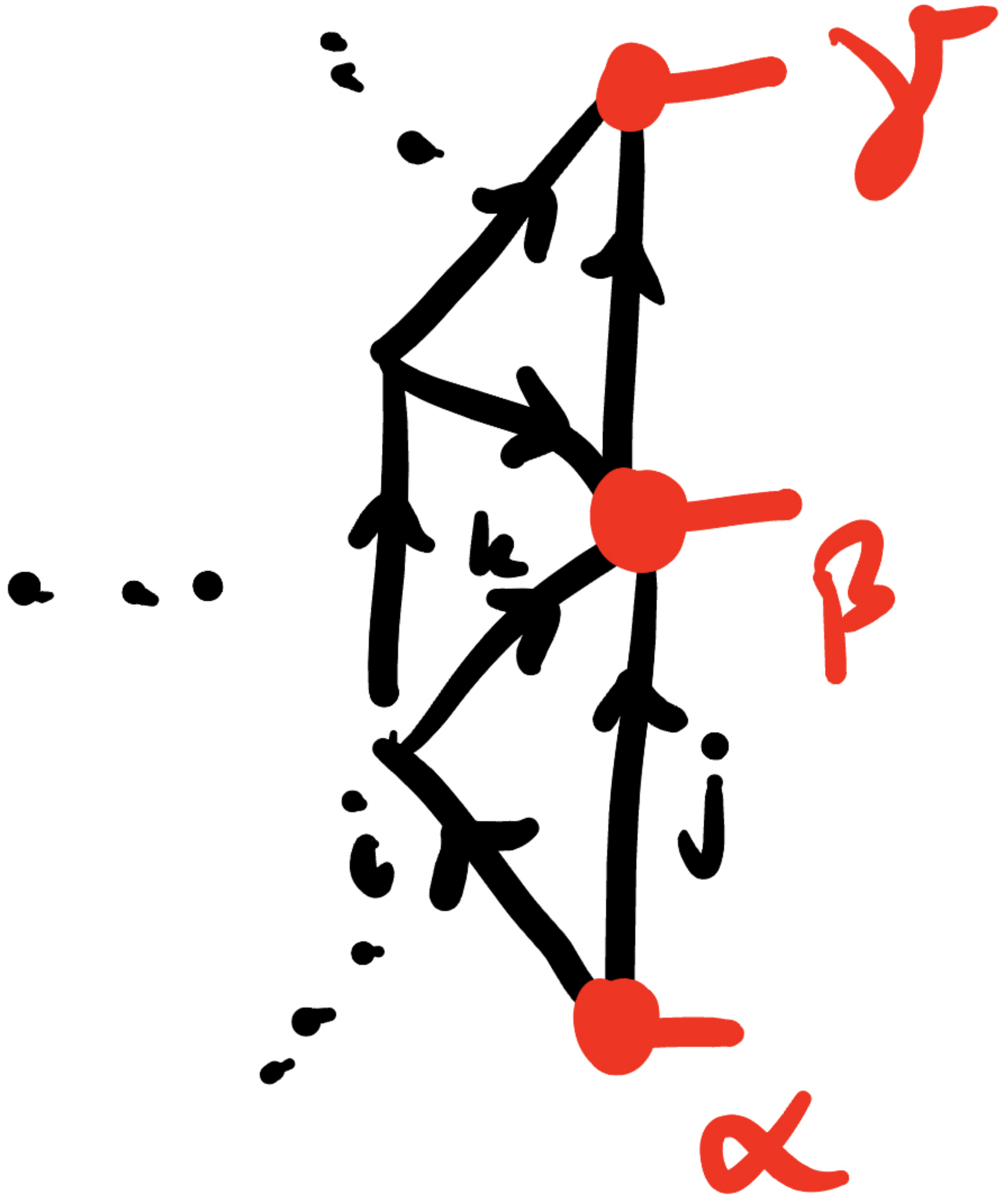
Example 3 *standard boundary*

the simples of any bulk model $\{1, i, j, k, \ldots\}$ **with**

$$i \triangleright j = i \times j \quad \forall i, j$$

define a consistent boundary model

$$i \triangleright \alpha = \bigoplus_{\beta} M_{i,\alpha}^{\beta}\, \beta, \quad M_{i,\alpha}^{\beta} \in \mathbb{Z}^{+}$$

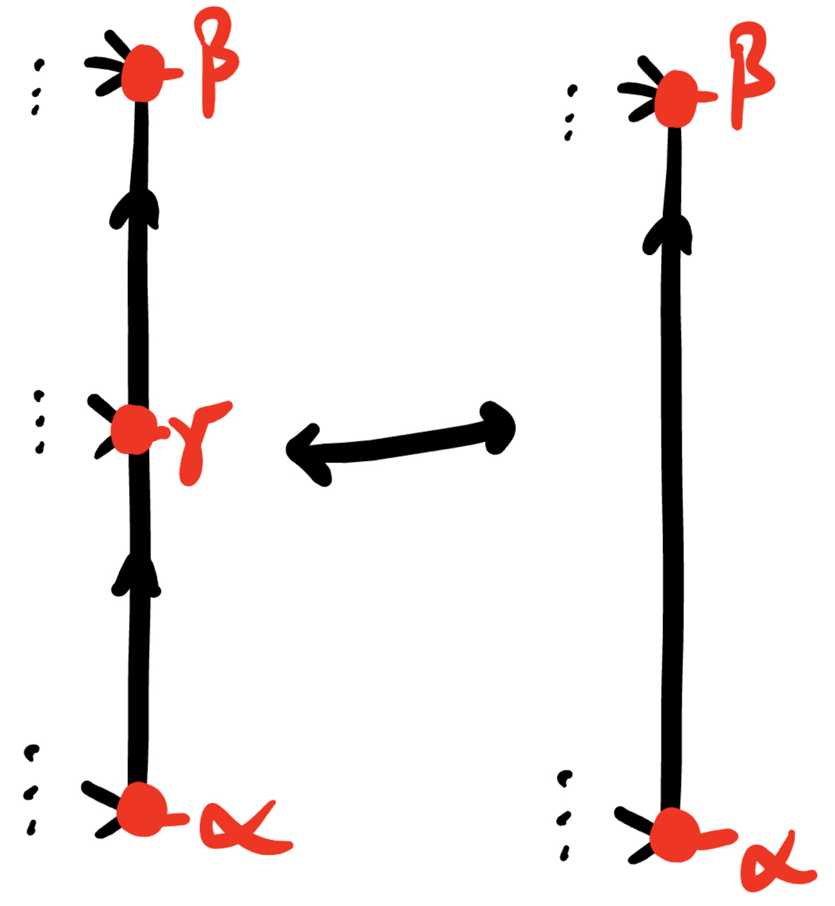


consistent with bulk N_{ij}^{k} $\quad i \triangleright (j \triangleright \alpha) \simeq (i \times j) \triangleright \alpha, \quad \forall i, j, \alpha$

fixed-point models

boundary

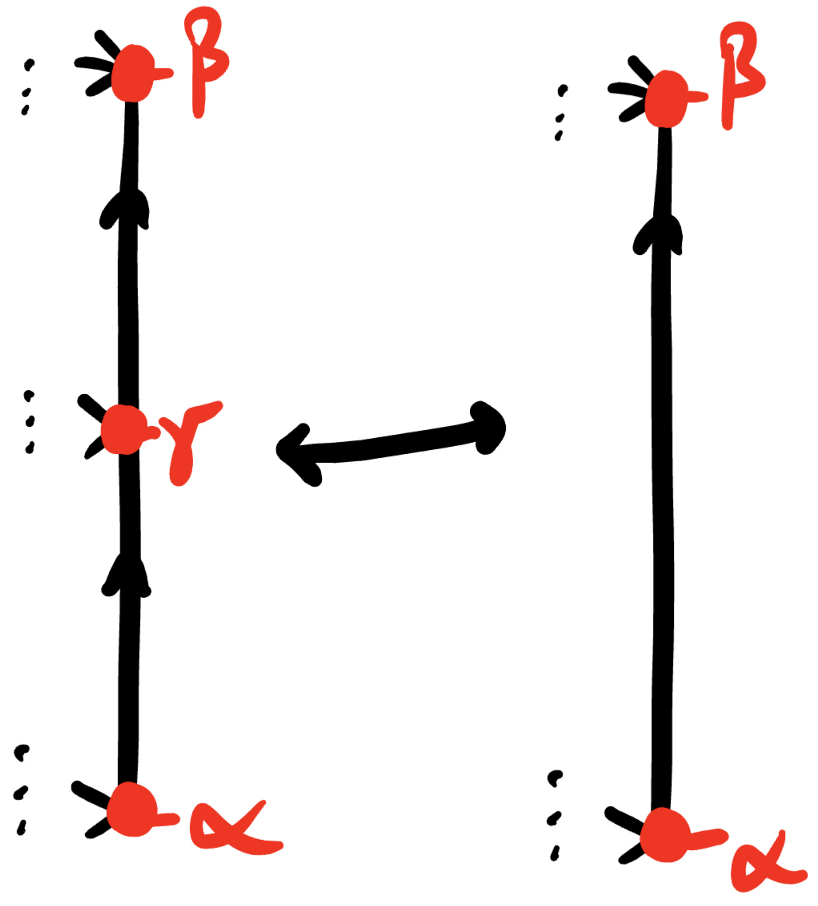
topological invariance by adding moves at boundary



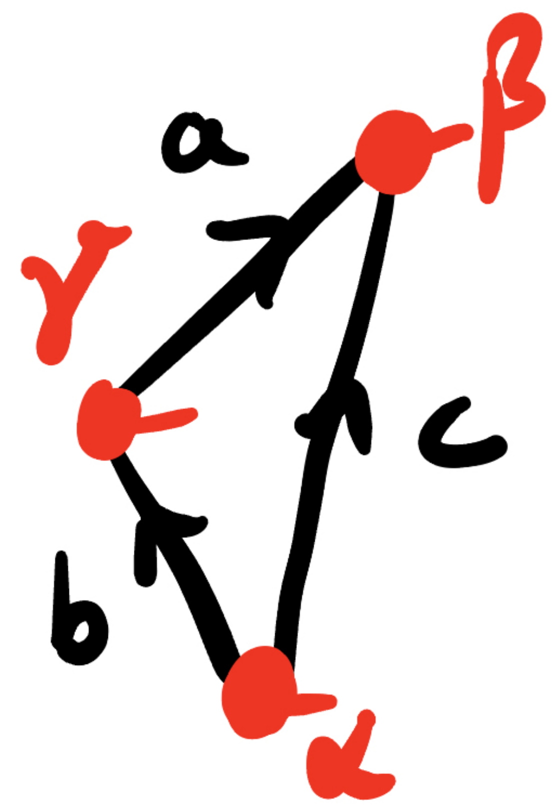
fixed-point models

boundary

topological invariance by adding moves at boundary



induce **linear maps**, represented by

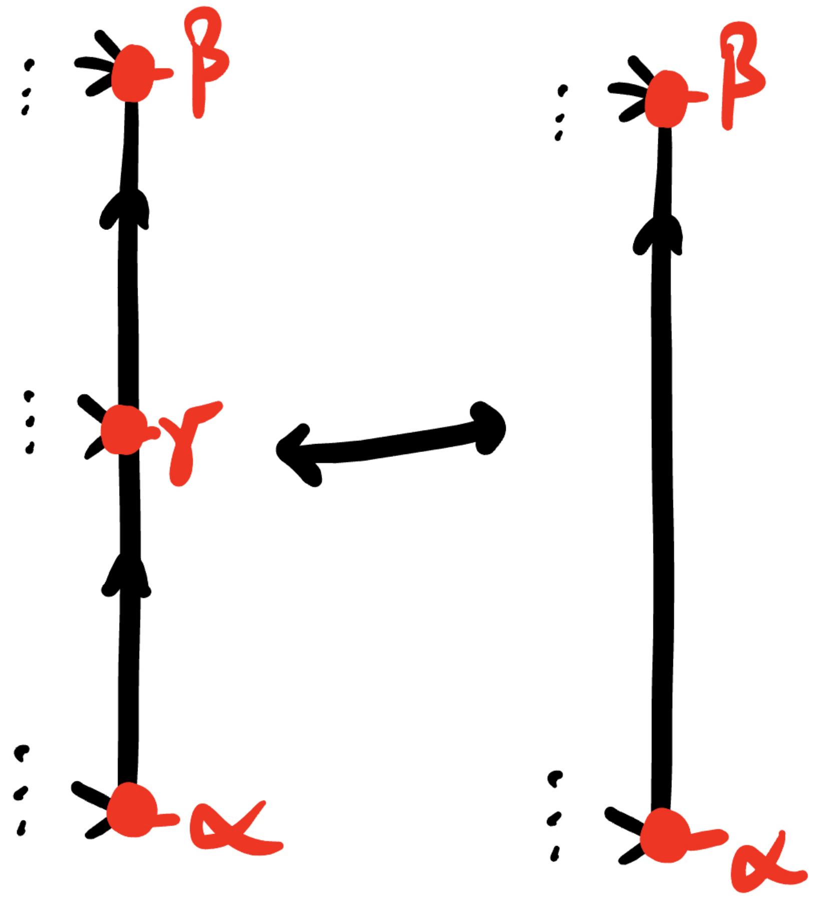


$$\rightsquigarrow \quad L^{\beta\gamma c}_{\alpha a b}$$

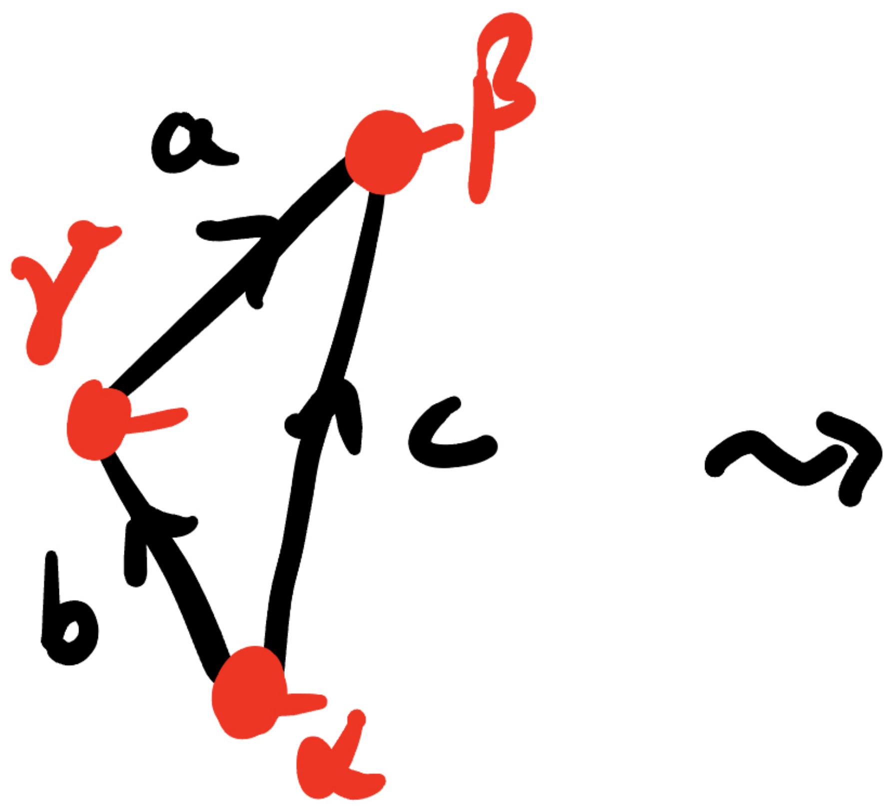
fixed-point models

boundary

topological invariance by adding moves at boundary



induce **linear maps**, represented by



$$L^{\beta\gamma c}_{\alpha ab}$$

equivalent sequences of moves

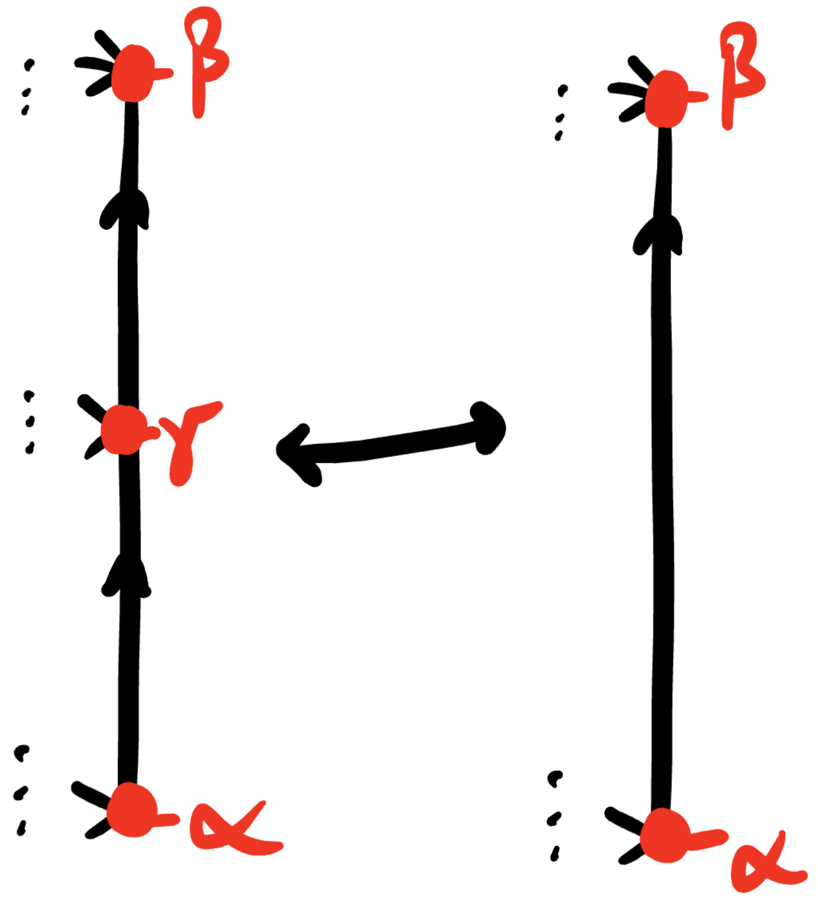
$$L^{\delta\beta d}_{\alpha ea} L^{\delta\gamma e}_{\beta cb} = \sum_f L^{\gamma\beta f}_{aba} L^{\delta\gamma d}_{\alpha cf} F^{baf}_{dce}$$

+ similar constraints for other branching structures

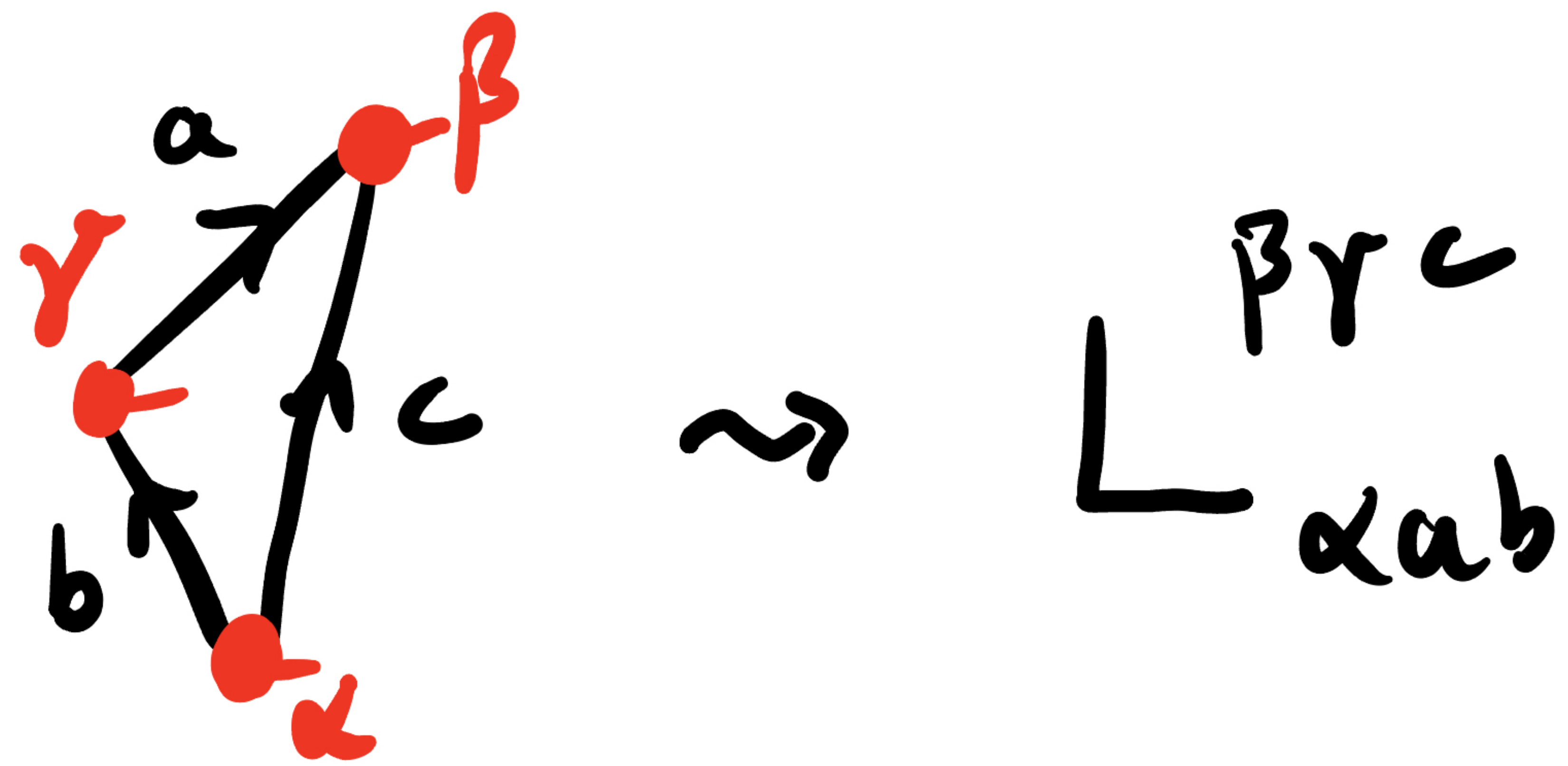
fixed-point models

boundary

topological invariance by adding moves at boundary



induce **linear maps**, represented by



! **equivalent sequences** of moves !

$$L^{\delta\beta d}_{\alpha e a} L^{\delta\gamma e}_{\beta c b} = \sum_f L^{\gamma\beta f}_{\alpha b a} L^{\delta\gamma d}_{\alpha c f} F^{baf}_{dce}$$

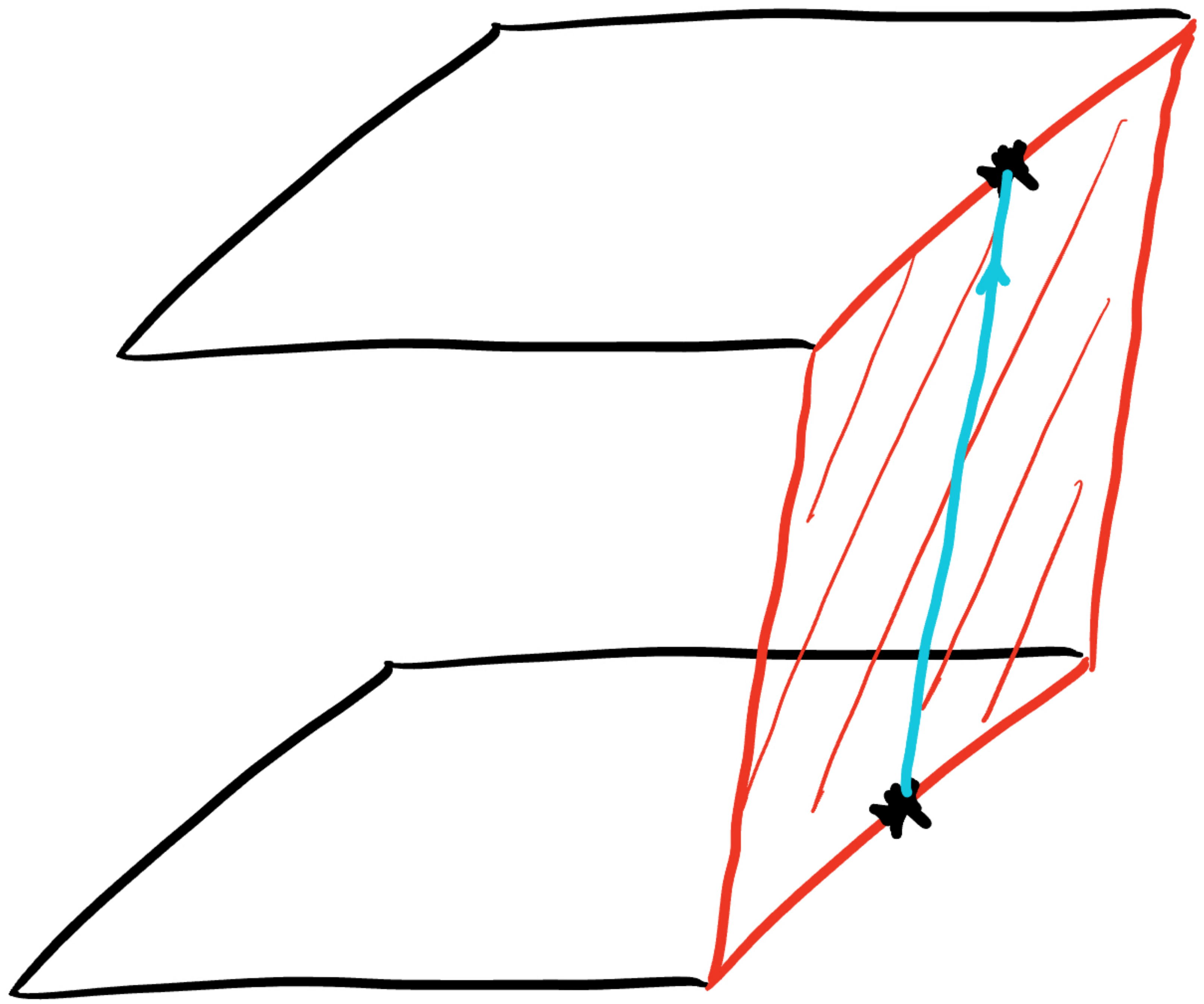
+ similar constraints for other branching structures

constraints on $\left\{ \{\alpha, \beta, \dots\}, \{M^{\beta}_{i\alpha}\}, \{L^{\beta\gamma k}_{\alpha ij}\} \right\}$ **for**

bulk $\mathscr{C}$ defines $\mathscr{C}$-module category ${}_{\mathscr{A}}\mathscr{M}$

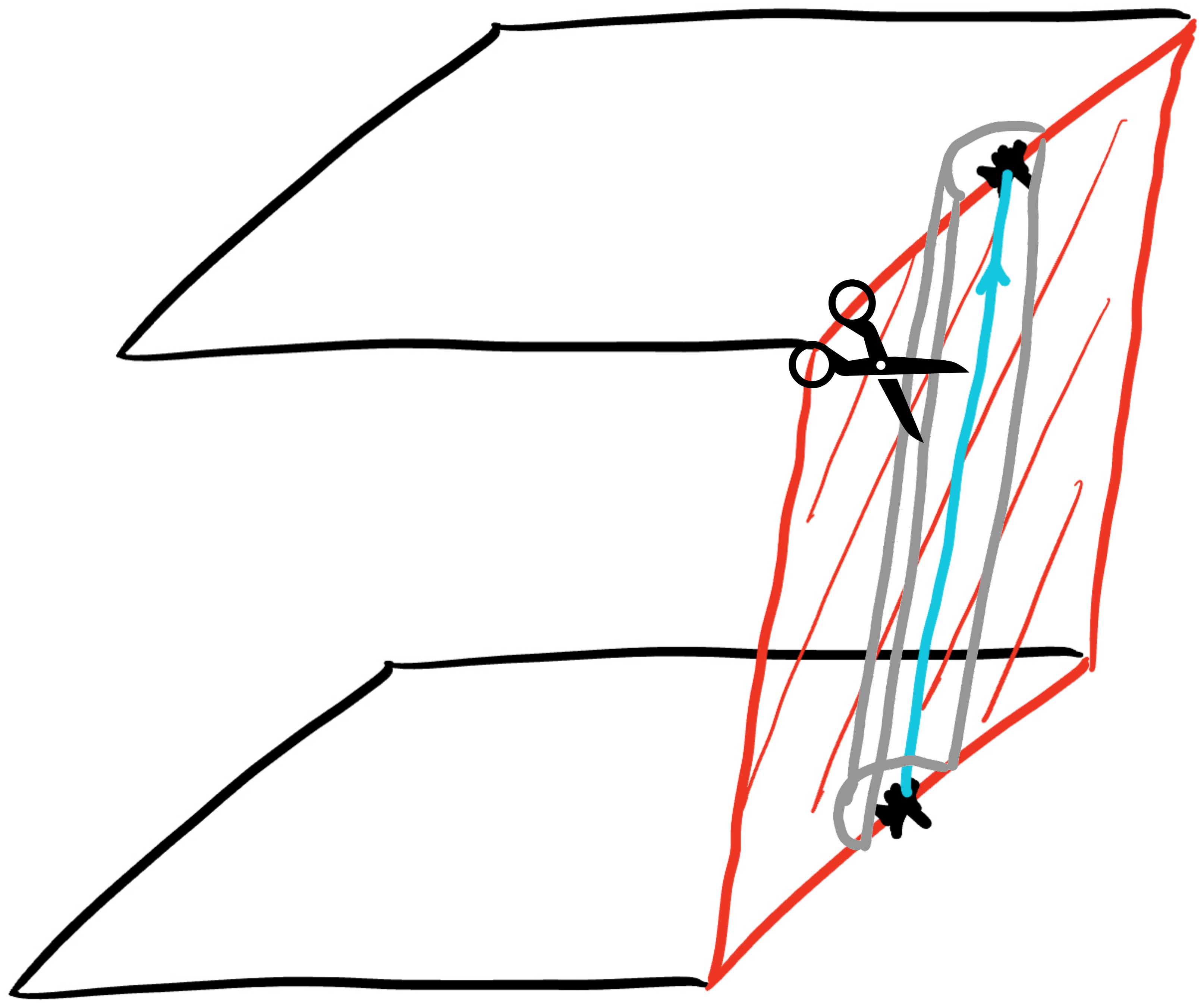
anyons at the boundary

semi-tube algebra



anyons at the boundary

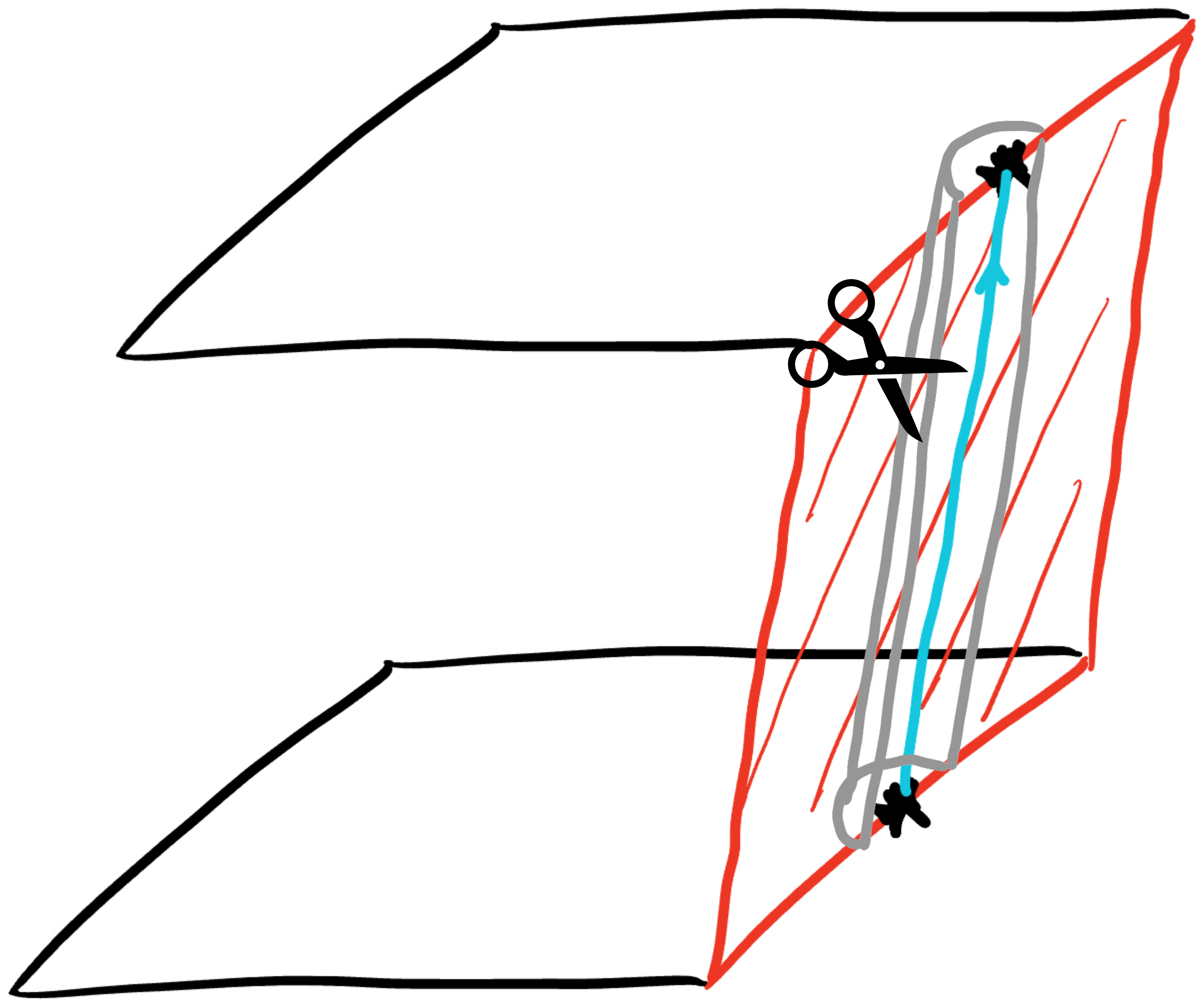
semi-tube algebra

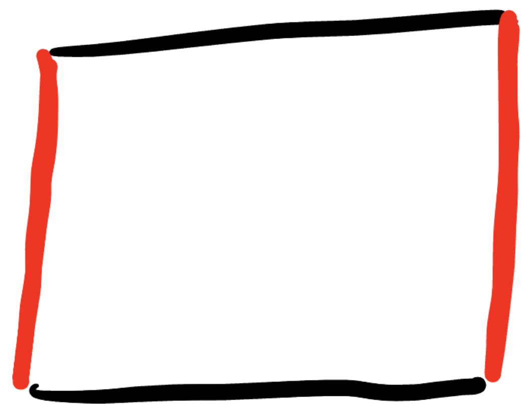


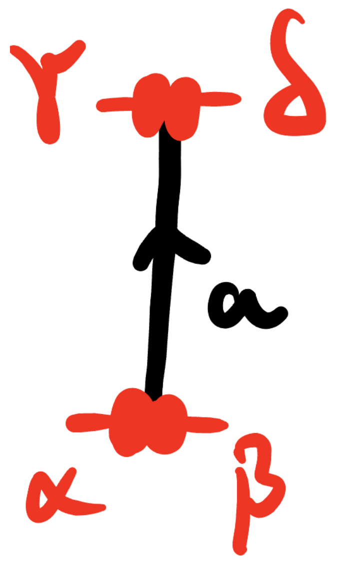
boundary line defect described by $[0,1] \times [0,1] =: S$

anyons at the boundary

semi-tube algebra



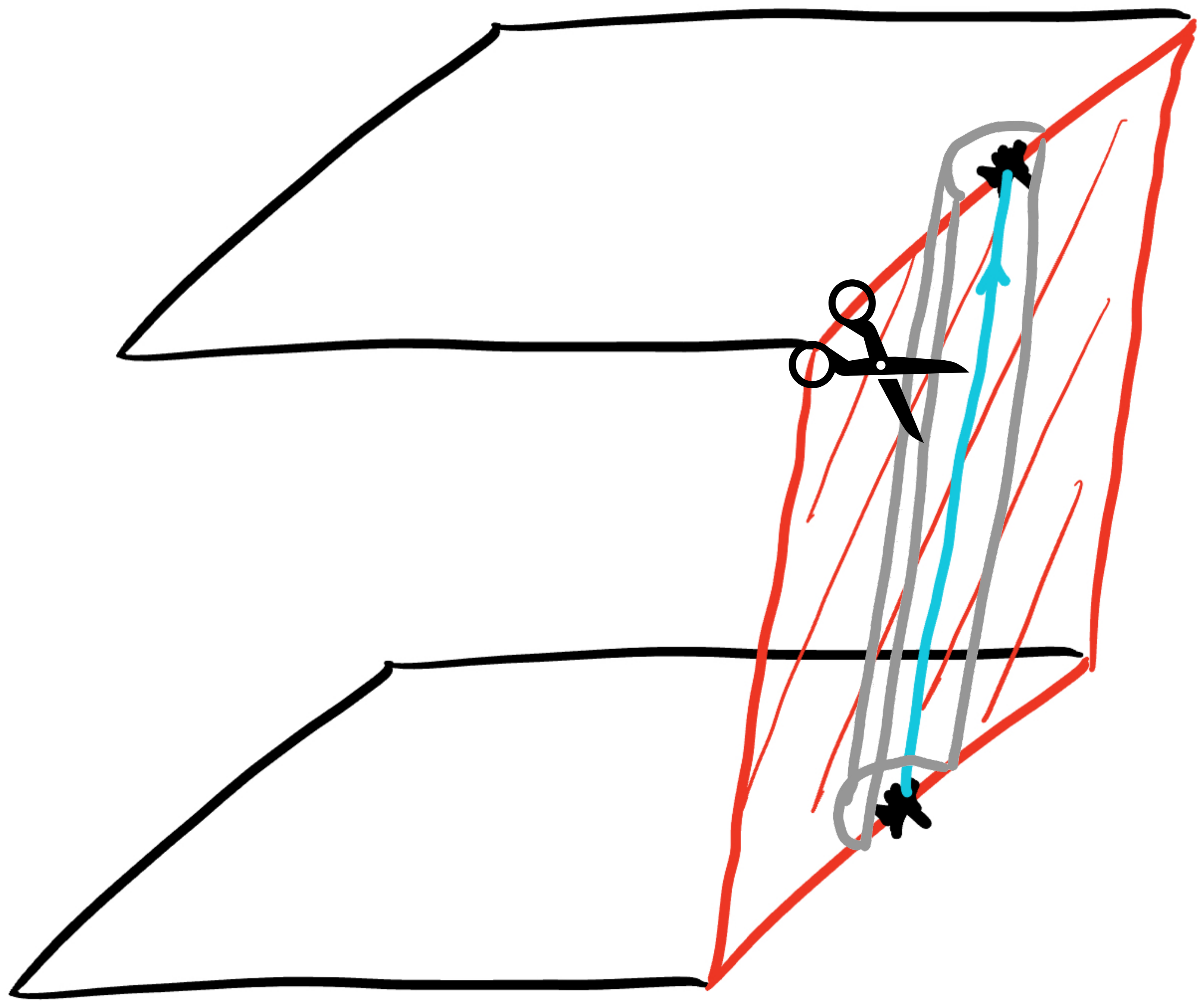
microscopic model:  $\mapsto V_S \simeq \mathbb{C}^{D_S}$

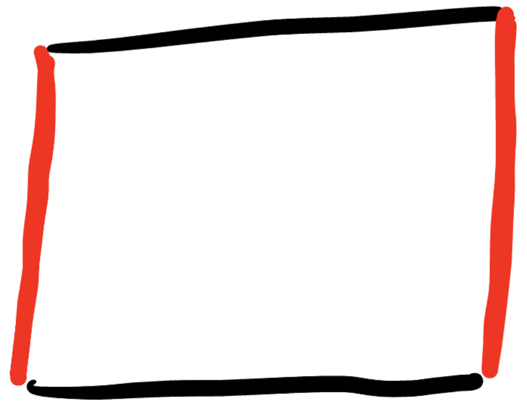
$V_S = \mathrm{span}_{\mathbb{C}} \Big\{$  $\Big\}$

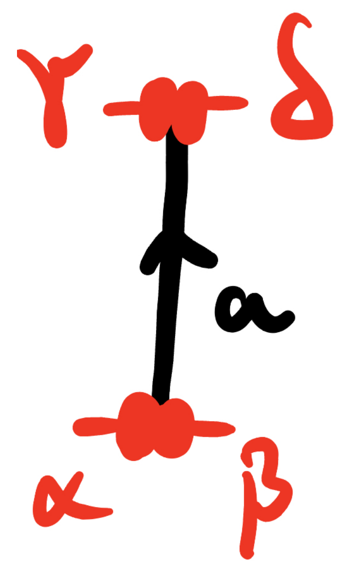
boundary line defect described by $[0,1] \times [0,1] =: S$

anyons at the boundary

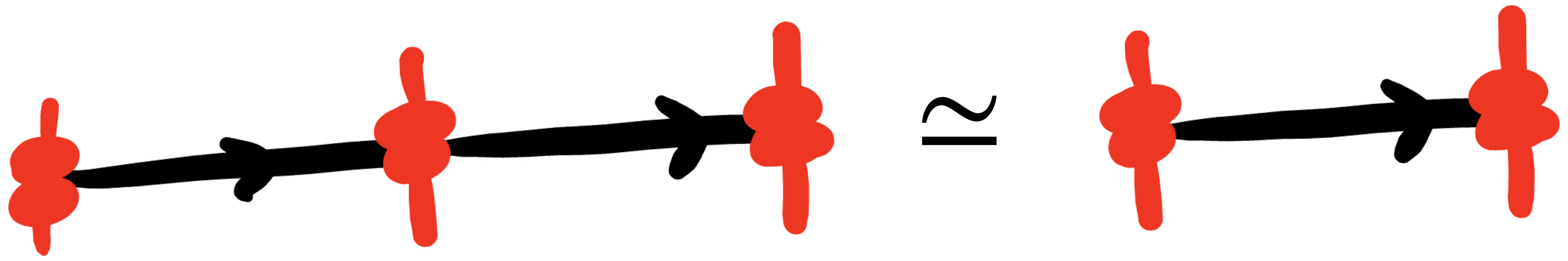
semi-tube algebra



microscopic model:  $\mapsto V_S \simeq \mathbb{C}^{D_S}$

$V_S = \mathrm{span}_{\mathbb{C}}$ {  }

V_S is not only a vector space…



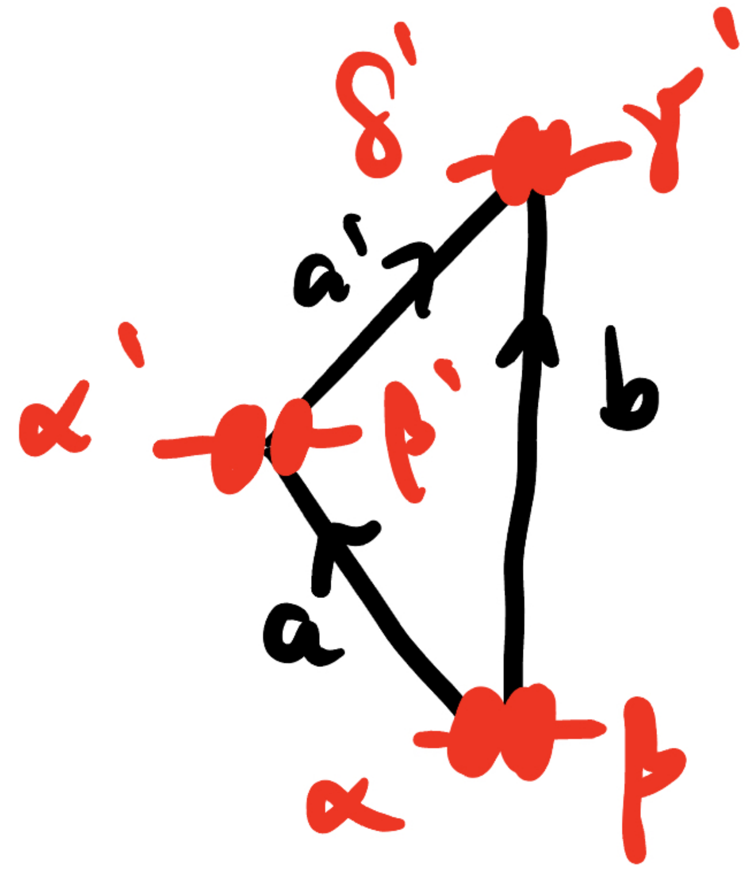
…but also an **algebra**

boundary line defect described by $[0,1] \times [0,1] =: S$

anyons at the boundary

semi-tube algebra

multiplication $*$ on V_S defined by **linear map** associated to

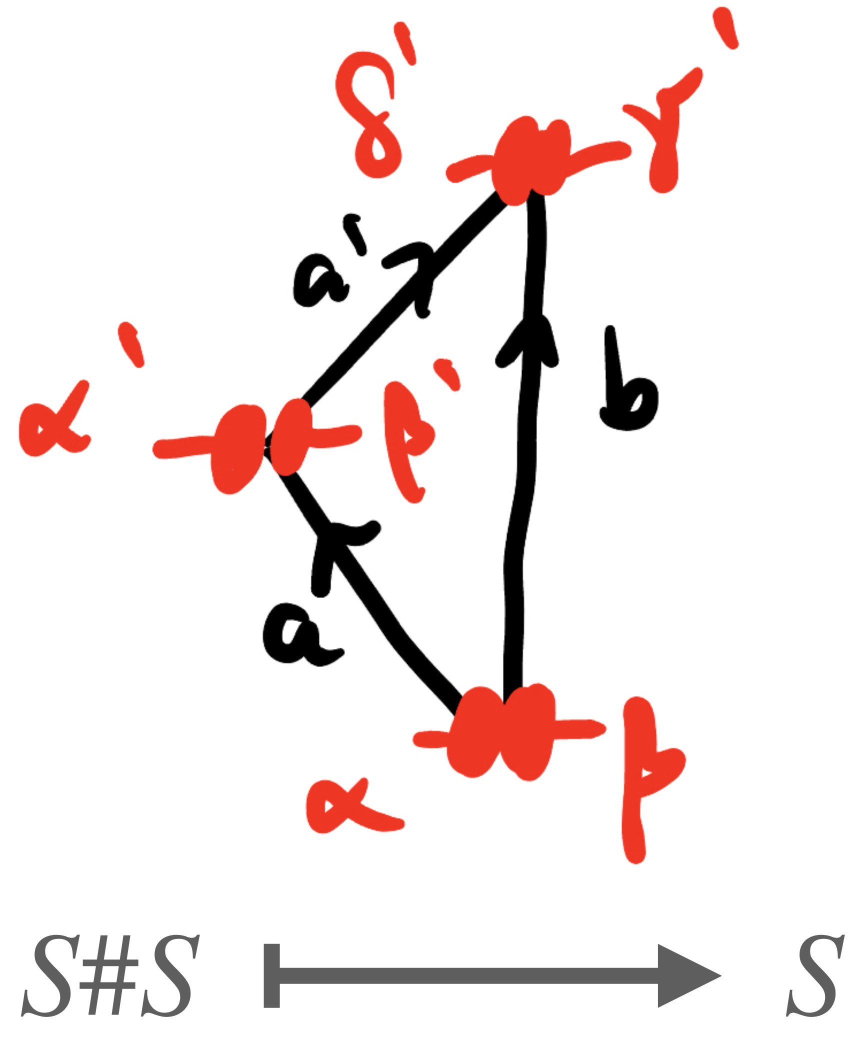


$S \# S \longmapsto S$

anyons at the boundary

semi-tube algebra

multiplication $*$ on V_S defined by **linear map** associated to



$$(\alpha',\beta',\gamma',\delta',a')_S * (\alpha,\beta,\gamma,\delta,a)_S = \delta_{\alpha',\delta}\delta_{\beta',\gamma}\sum_b L^{\gamma'\beta' b}_{\beta a a'}\overline{L^{\delta'\alpha' b}_{\alpha a a'}}(\alpha,\beta,\delta',\gamma',b)_S$$

as an algebra, $V_S = \bigoplus_j S_j$ with S_j irreducible subspaces

associated to **central idempotents**

anyons at the boundary

semi-tube algebra

Example 1'

for bulk Vec(G), a subgroup H together with a 2-cocycle $\psi : H^2 \to U(1)$ fulfilling

$$\psi(a,b)\psi(ab,c) = \psi(a,bc)\psi(b,c)$$

$\alpha, \beta \in G/H$

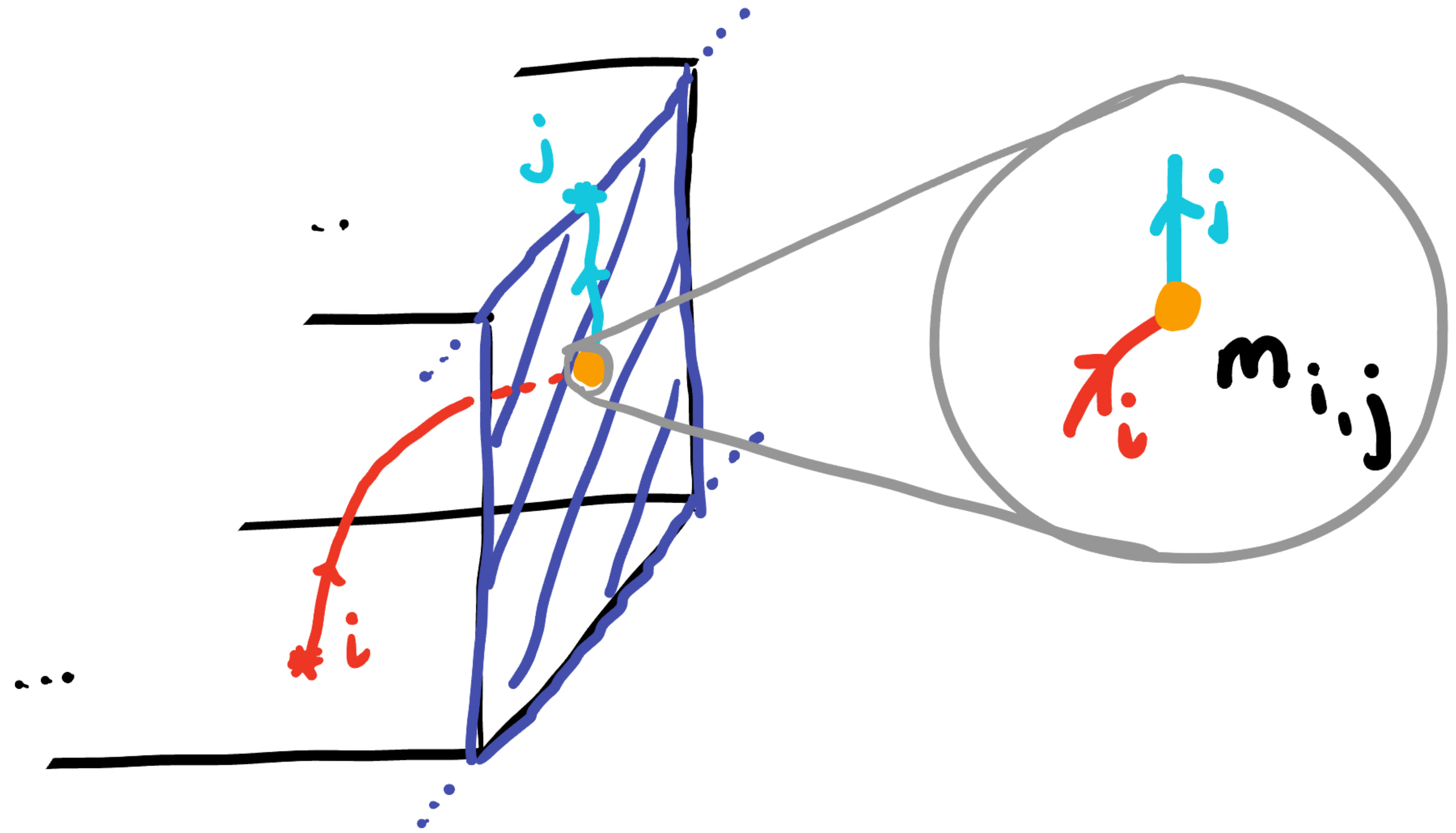
define valid boundary. Associated central idempotents of $S = \text{span}_{\mathbb{C}}\{(\alpha,\beta,g)_S\}$ are

$$c^S_{(x,\kappa_x)} = \frac{\dim(\kappa_x)}{|K_x|} \sum_{\substack{\alpha,\beta \in G/H \\ \alpha^{-1}\beta = x}} \sum_{g \in \text{Stab}_G((\alpha,\beta))} \overline{\tilde{\chi}^{(\alpha,\beta)}_{\kappa_x}(g)} (\alpha,\beta,g)_S$$

double coset: x

projective *Irrep of stabilizer group Stab*(x): κ_c

bulk-to-boundary fusion vertex



i : Irrep $T_i \subseteq V_T$
j : Irrep $S_j \subseteq V_S$

$m_{i,j}$: multiplicity of $T_i \otimes S_j \subseteq V_C$

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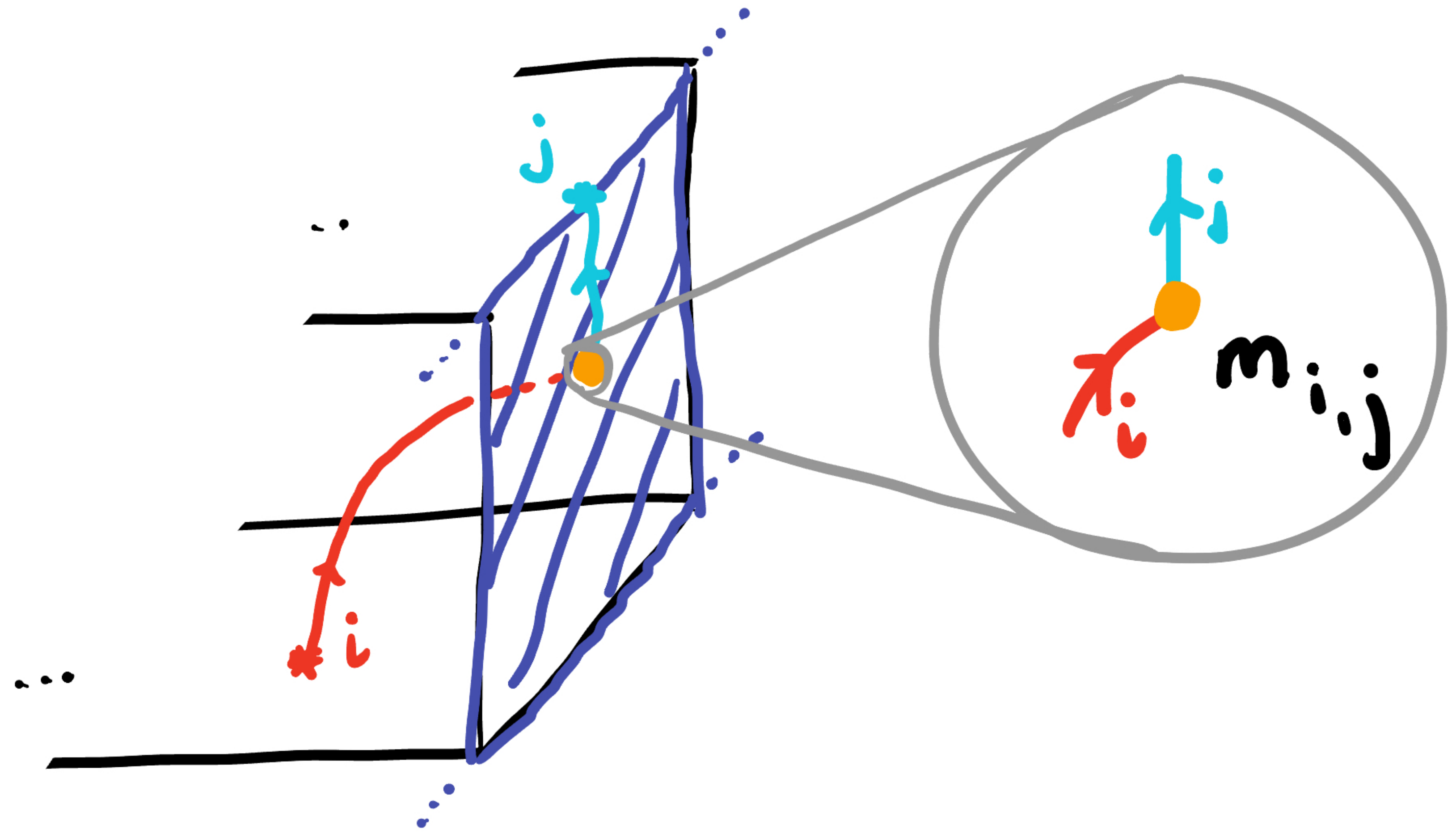

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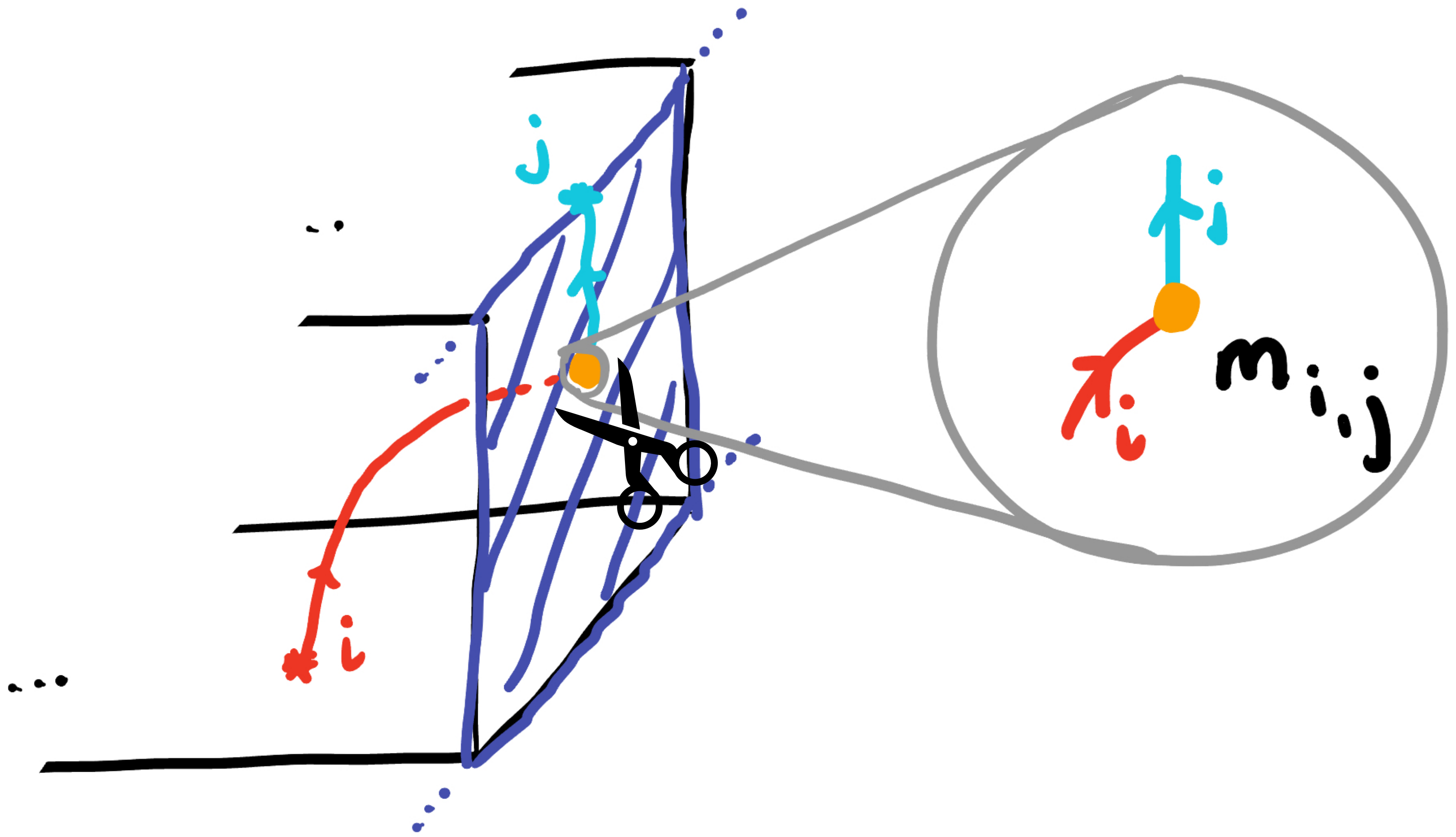

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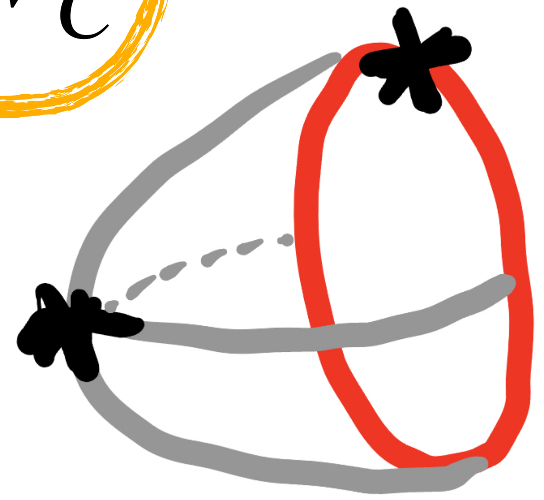
fusion vertex is described by 

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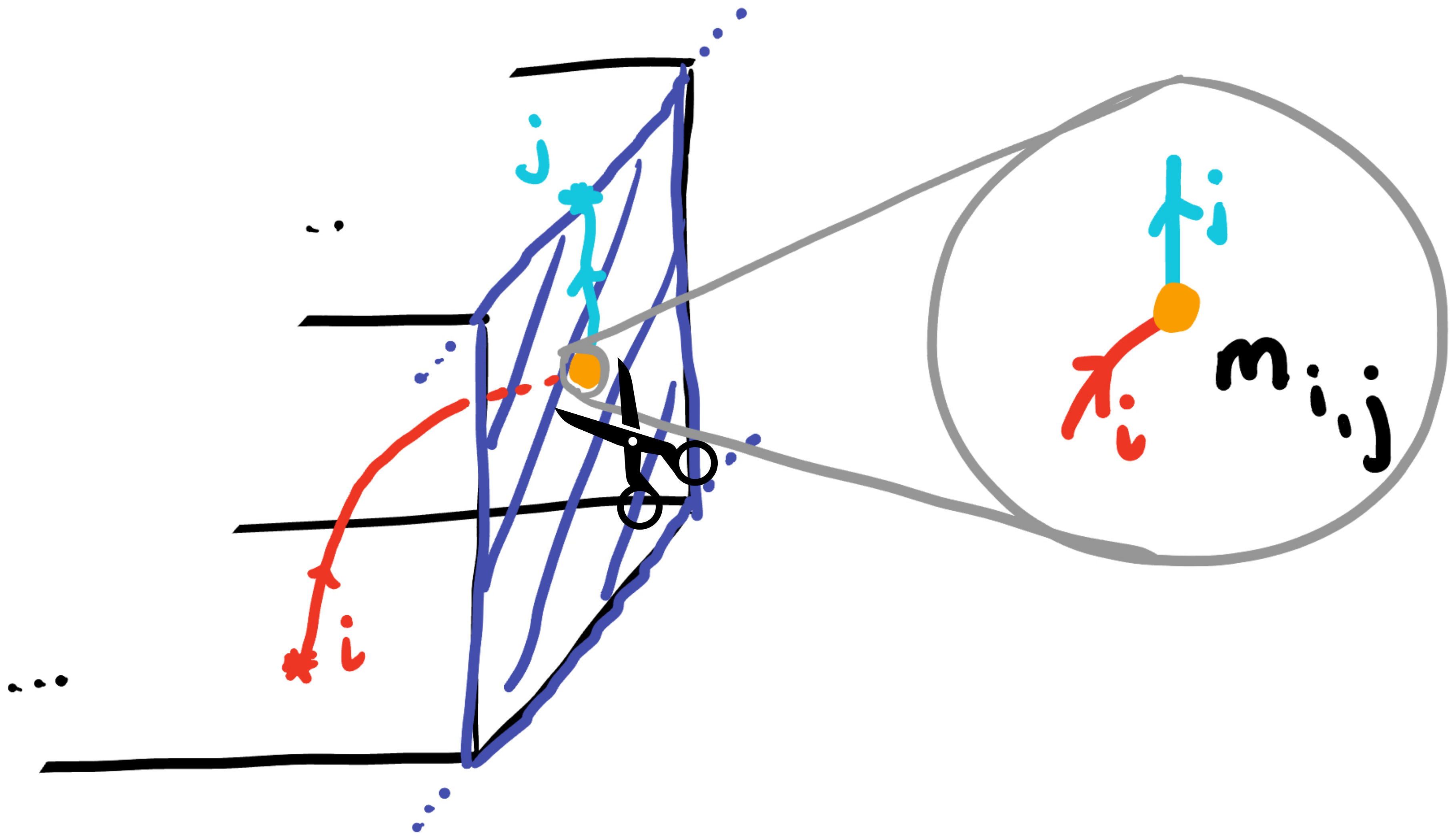

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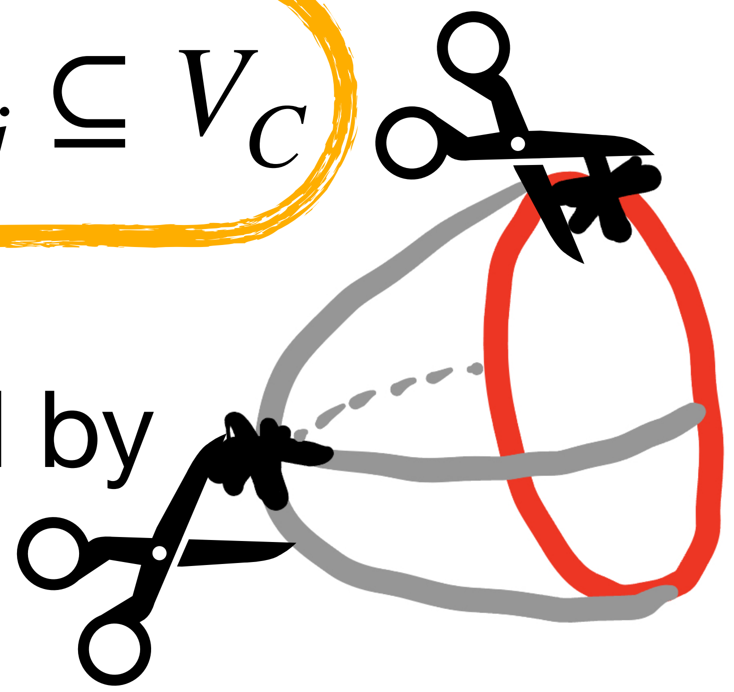

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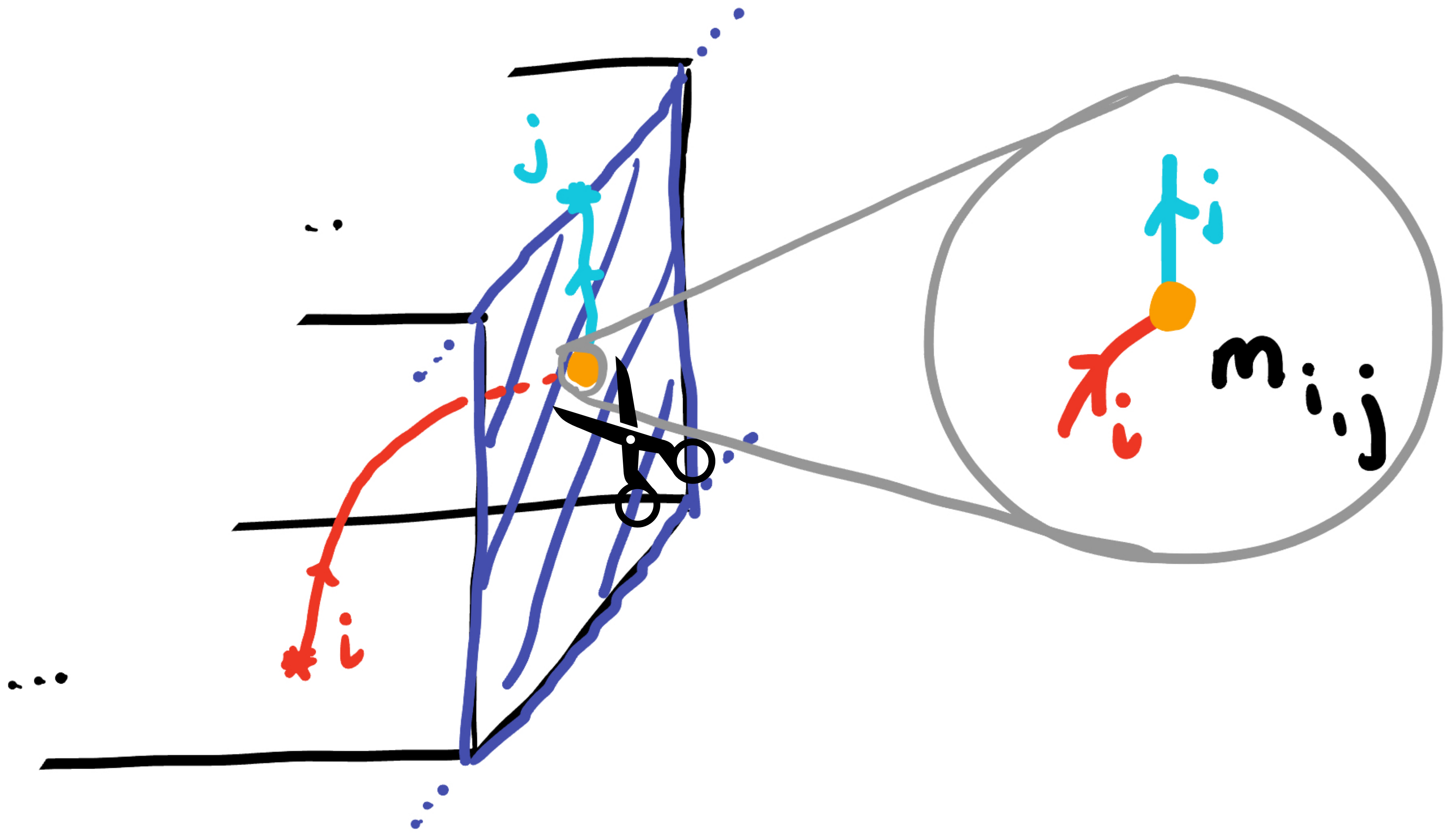
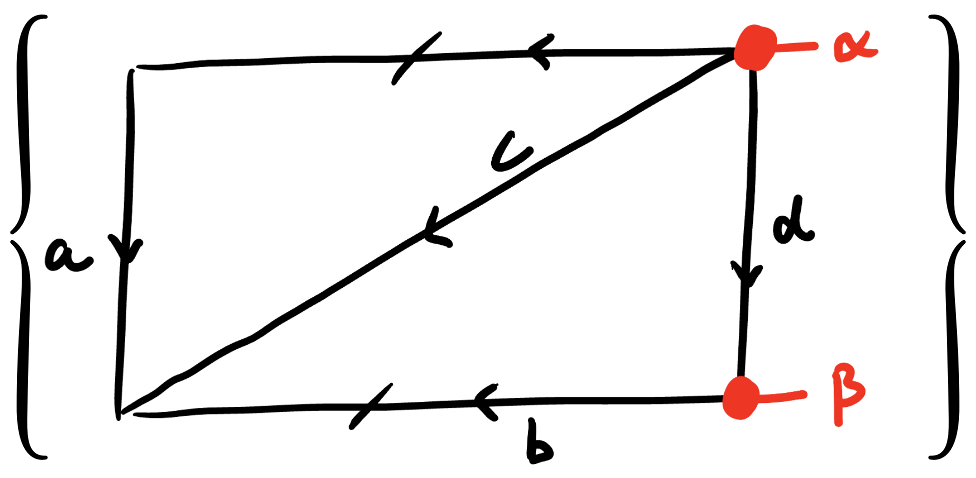
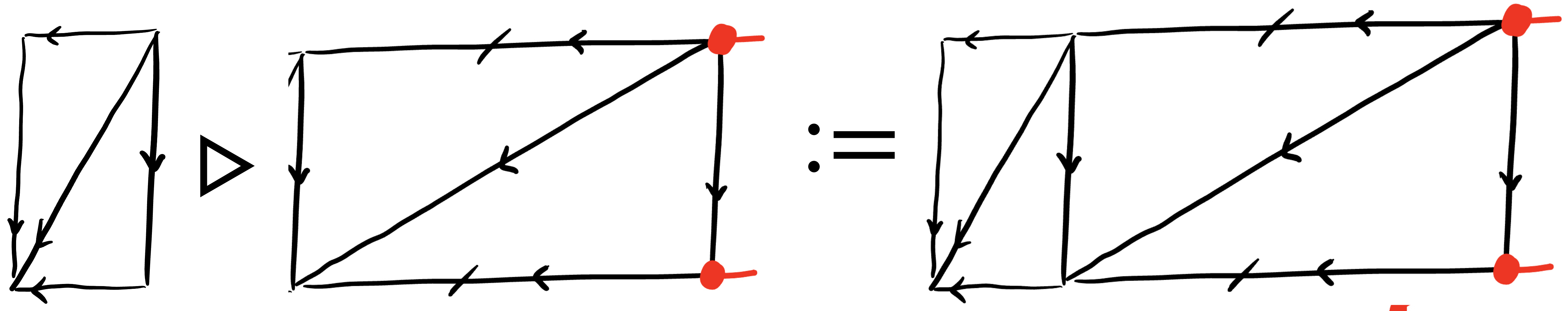
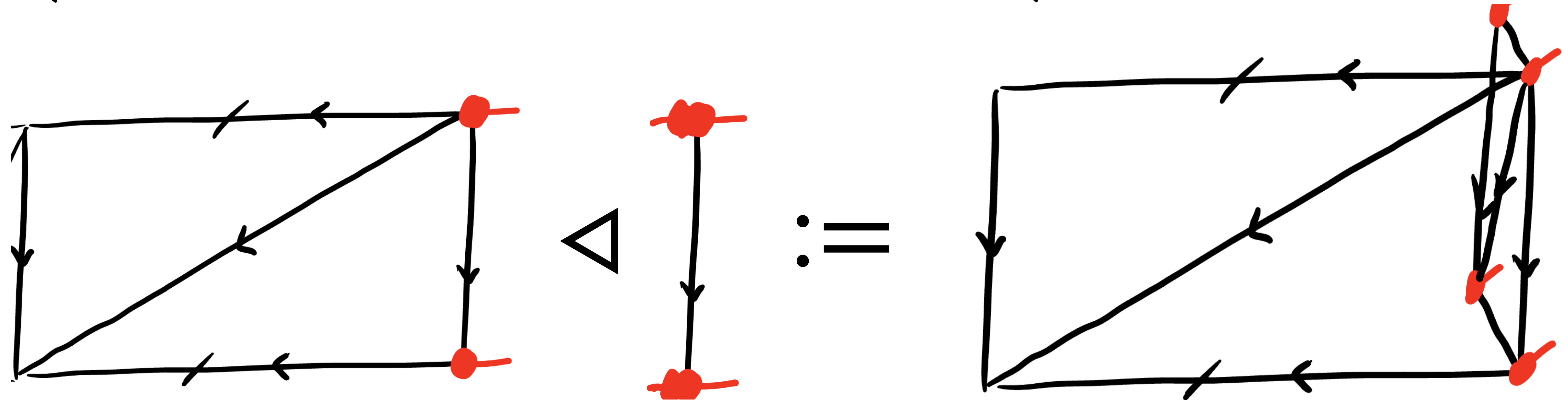


We get space

$V_C = \mathrm{span}_{\mathbb{C}}$ 

with $V_T \triangleright$ and $\triangleleft V_S$ action,





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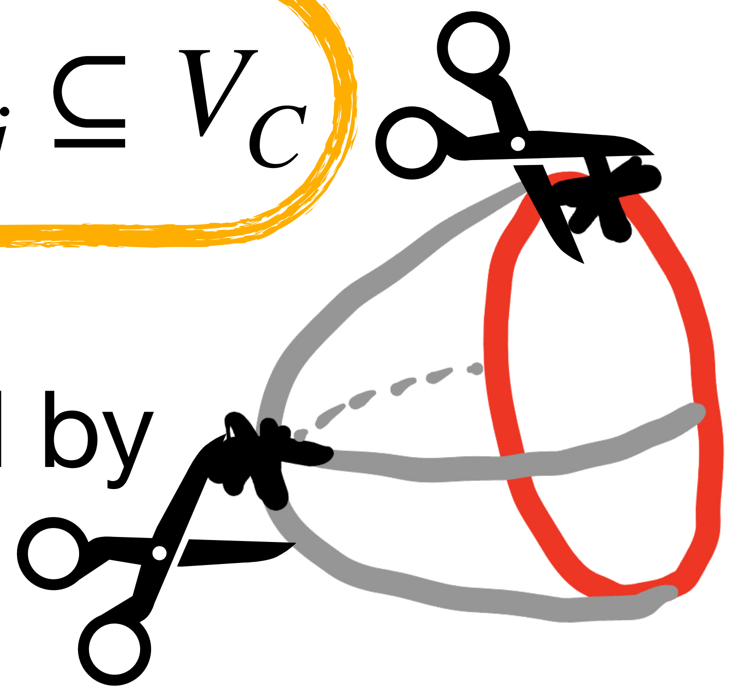

bulk-to-boundary fusion multiplicities

actions $\triangleright$ and $\triangleleft$ commute

"bi-representation" of algebras V_T and V_S

$$V_C = \bigoplus_{i,j} m_{i,j} T_i \otimes S_j$$

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using **central idempotents**, we can **project onto** $m_{i,j} T_i \otimes S_j$

$$\boxed{m_{i,j} = \frac{1}{\text{dim}(T_i)\text{dim}(S_j)} \text{Tr}\left(c_i^T \triangleright \bullet \triangleleft c_j^S\right)}$$

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Example 1: Vec$^{\omega}(G)$ with valid subgroup H and 2-cocycle ψ

$$m_{(c,\rho_c),(x,\kappa_x)} = \frac{1}{|G|} \sum_{g \in c} \sum_{\substack{\alpha \in G/H \\ \alpha^{-1} g^{-1} \alpha = x}} \sum_{h \in Z(g) \cap \mathrm{Stab}_G((g \triangleright \alpha, \alpha))} \psi^{\alpha}(h, g) \overline{\psi^{\alpha}(g, h)} \tilde{\chi}^{g}_{\rho_c}(h) \overline{\tilde{\chi}^{(g \triangleright \alpha, \alpha)}_{\kappa_x}(h)}$$

applications

TQFTs and topological quantum error correction

- explicit construction of one more invariant for TQFTs with microscopic models
- restriction on **trivial boundary anyon** gives **Lagrangian algebra object**
    - *algebra morphism* calculated together with bulk anyon fusion vertex

- describing and designing **QEC protocols** based on topological codes **with defects**
    - code dimension
    - logical algebra
    - folding trick: interfaces between codes
    - …

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more ideas?

conclusion and outlook

- **microscopic models** for non-chiral topological theories with boundaries
- description of anyons in terms of **tube algebra**
- construction of **bi-representation** describing **bulk-boundary fusion vertex**
- explicit formula for Vec$^{\omega}(G)$ models for **bulk-boundary fusion multiplicities**

- describe existing protocol(s) involving **twisted** gauge theory models (ongoing)
- include **algebra morphism** for condensable object (ongoing)
- resolving similar fusion vertex of **line to line-on-surface defect** in 3+1d
- understand **more general defects in 3+1d** with microscopic model

arXiv:2302.01835

Thank you!

Questions?