Internal Levin–Wen models

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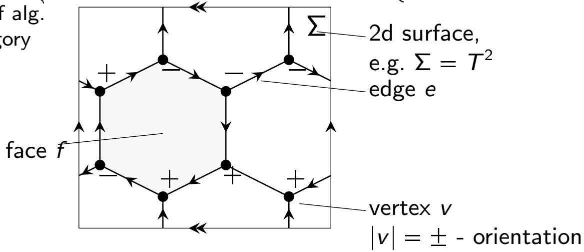
Idea

Kitaev and Levin–Wen models give (some of the) topological phases.
Both work similarly:

$$\underbrace{\text{algebraic input } \mathbb{A},}_{\substack{\text{Kitaev – s.si. fin. dim. Hopf alg.} \\ \text{LW – spherical fusion category}}} \quad \text{surface } \Sigma, \quad \text{lattice } \Gamma \subseteq \Sigma \quad \rightsquigarrow \quad \begin{cases} \text{state space } V \\ \text{Hamiltonian } H \end{cases}$$

e.g.



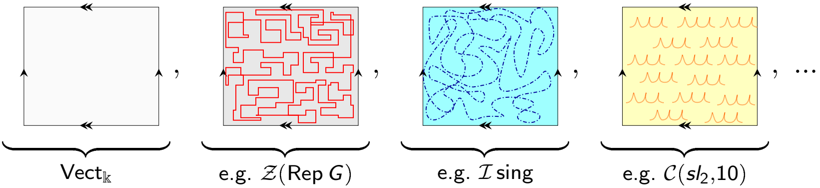
$\mathbb{A}$ yields a vector space X, one sets $V = \bigotimes_{v \in \Gamma_0} X^{|v|}$ and the Hamiltonian

$$H = \sum_{v \in \Gamma_0} (\mathrm{id}_V - P_v) + \sum_{e \in \Gamma_1} (\mathrm{id}_V - P_e) + \sum_{f \in \Gamma_2} (\mathrm{id}_V - P_f)$$

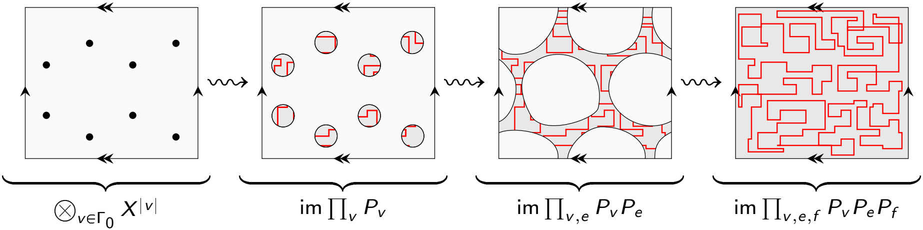
where $\quad P_c P_c = P_c, \quad [P_c, P_{\tilde{c}}] = 0 \quad$ – commuting projectors.

Idea

Imagine the numerous topological phases that can live on a surface Σ:



Kitaev $\subseteq$ Levin–Wen models gradually project onto the top. phase:



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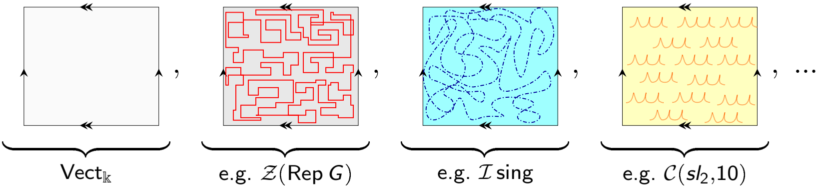
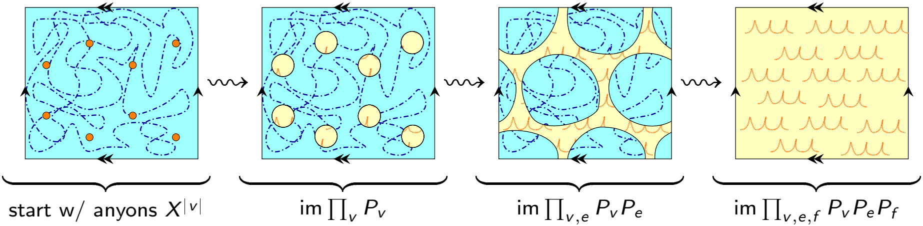


Internal Levin–Wen models gradually project a phase $\mathcal{C}$ onto a phase $\mathcal{D}$:



Prerequisites

To define internal Levin–Wen models, we will need:

1. Reshetikhin–Turaev (RT) TQFTs with embedded ribbon graphs (a.k.a. graph TQFTs or TQFTs with line and point defects)
2. A generalisation of RT TQFTs which includes surface defects via internal (2d) state-sum
3. Internal (3d) state-sum or generalised orbifold construction on RT TQFTs with line and surface defects

In short:

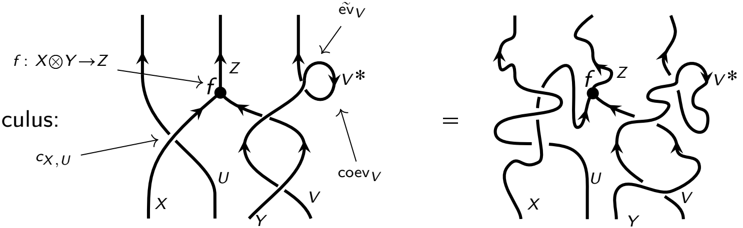
graph TQFT $Z_{\mathcal{C}}^{\mathrm{RT}} \xrightarrow{\text{int. 2d ss}}$ defect TQFT $Z_{\mathcal{C}}^{\mathrm{def}} \xrightarrow{\text{int. 3d ss}}$ orbifold TQFT $Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}$

Modular categories and anyons

Modular fusion category $\mathcal{C}$ is

- finitely semisimple:
  - $\mathcal{C}(X,Y) := \mathrm{Hom}_{\mathcal{C}}(X,Y)$ – $\Bbbk$-vector space,
  - U – simple $\Leftrightarrow$ $\mathcal{C}(U,U) \cong \Bbbk$,
  - $\mathrm{Irr}_{\mathcal{C}} = \{U_i \text{ – simple}\}$ is finite,
  - $X \cong \bigoplus_i U_i^{\oplus N_X^i}$
- fusion: monoidal with $\otimes$ linear on hom.'s
- braided: $c_{X,Y} \colon X \otimes Y \to Y \otimes X$
- spherical: $\mathrm{ev}_X \colon X^* \otimes X \to \mathbb{1}$, $\mathrm{coev}_X \colon \mathbb{1} \to X \otimes X^*$,
  $\widetilde{\mathrm{ev}}_X \colon X \otimes X^* \to \mathbb{1}$, $\widetilde{\mathrm{coev}}_X \colon \mathbb{1} \to X^* \otimes X$, left trace = right trace
- modular: transparent obj.'s are $\mathbb{1}^{\oplus n}$ (i.e. Müger centre is $\mathrm{Vect}_{\Bbbk}$)
- e.g. $\mathrm{Vect}_{\Bbbk}$, $\mathcal{Z}(\mathrm{Rep}\, G)$, $\mathcal{Z}(\mathrm{Rep}\, H)$, $\mathcal{Z}(\mathcal{S})$, ..., Vect_A^q, $\mathcal{F}\mathrm{ib}$, $\mathcal{I}\mathrm{sing}$, ...

Graphical calculus:

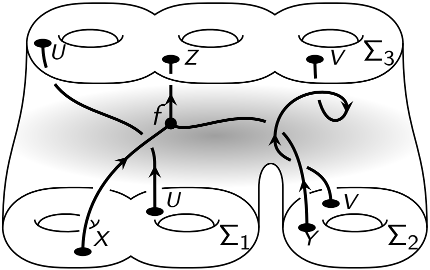


Modular categories and anyons

A modular fusion category $\mathcal{C}$ describes

point excitations $\Leftrightarrow$ **line and point defects** $\Leftrightarrow$ **anyons**

in a $3 = (2+1)$-dim. TQFT:



$$\underbrace{Z\colon \mathrm{Bord}_{3,2} \longrightarrow \mathrm{Vect}_{\Bbbk}}_{\text{ordinary TQFT}} \quad \rightsquigarrow \quad \underbrace{Z_{\mathcal{C}}\colon \mathrm{Bord}^{\mathrm{rib}}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\Bbbk}}_{\text{“}graph\,TQFT\text{''}}$$

Reference example: Reshetikhin–Turaev construction

Reshetikhin-Turaev graph TQFT

[Reshetikhin-Turaev'91]

... is the symmetric monoidal functor

$$Z^{\mathrm{RT}}_{\mathcal{C}} : \underbrace{\widehat{\mathrm{Bord}}^{\mathrm{rib}}_3(\mathcal{C})}_{\text{3d bordisms w/ } \mathcal{C}\text{-ribbon graphs - hence "graph TQFT"}} \longrightarrow \mathrm{Vect}$$

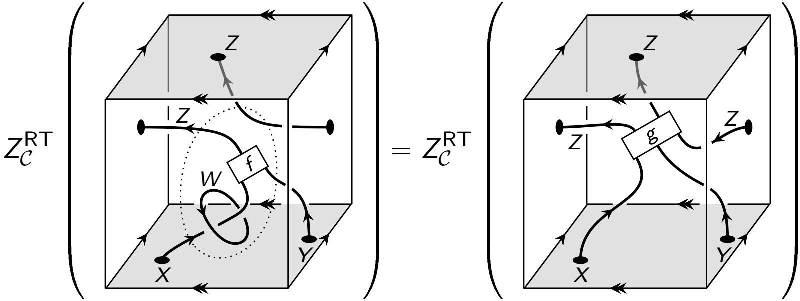
↓ signature extension - to eliminate gluing anomaly

obtained from a modular fusion category (MFC) $\mathcal{C}$ + a choice for $\sqrt{\mathrm{Dim}\,\mathcal{C}}$

[Turaev'94]

by applying the universal construction on the RT invariants of closed 3d manifolds with embedded $\mathcal{C}$-coloured ribbon graphs.

[Blanchet et al.'95], [De Renzi et al.'19]



Reshetikhin-Turaev defect TQFT

[Kapustin-Saulina'11]
[Fuchs-Schweigert-Valentino'13]
[Carqueville-Runkel-Schaumann'19]

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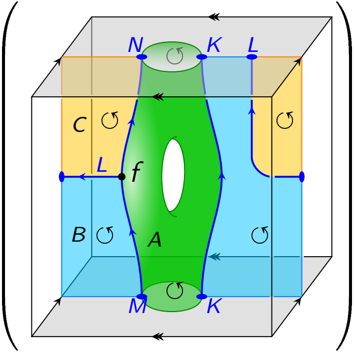
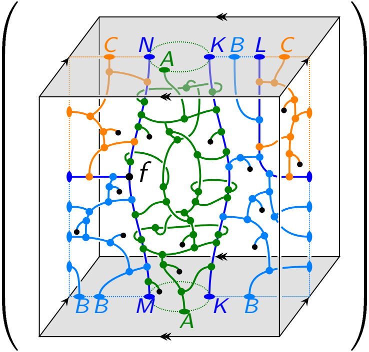
$$Z^{\mathrm{def}}_{\mathcal{C}} : \underbrace{\widehat{\mathrm{Bord}}^{\mathrm{def}}_3}_{\text{stratified 3d bordisms}}(\overset{\downarrow \text{'defect datum' - labels and adjacency data for strata}}{\mathbb{D}^{\mathcal{C}}}) \longrightarrow \mathrm{Vect}$$

defined i.t.o. $Z^{\mathrm{RT}}_{\mathcal{C}}$ via an internal 2d state-sum construction, e.g.

$Z^{\mathrm{def}}_{\mathcal{C}}$



$:= Z^{\mathrm{RT}}_{\mathcal{C}}$



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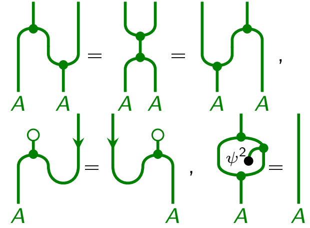
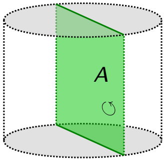
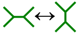
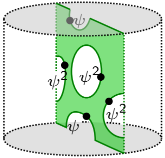
Surfaces are labelled by symmetric separable Frobenius algebras $A \in \mathcal{C}$:

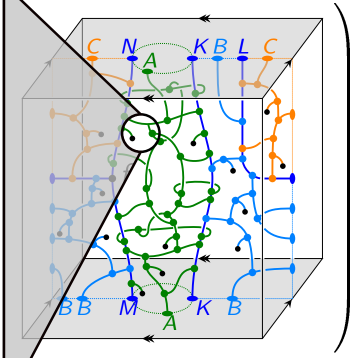




the conditions mimic the moves  and  on 1-skeleta (e.g. duals of triangulations)...



...or equivalently allow to punch holes



Reshetikhin-Turaev defect TQFT

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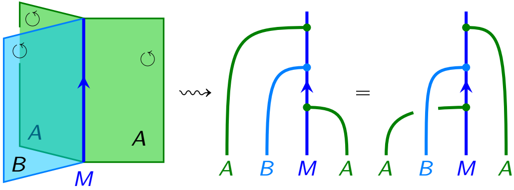
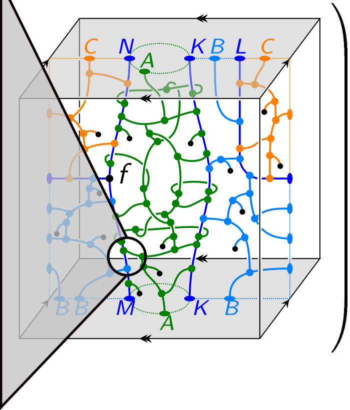
defined i.t.o. $Z^{\mathrm{RT}}_{\mathcal{C}}$ via an internal 2d state-sum construction, e.g.

Lines are labelled by multimodules of ssFA's assigned to adjacent surfaces, e.g.



where M has right A and left A, B actions which commute in the same way the adjacent surfaces overlap (equivalently, M is an $(A \otimes B)$-A-bimodule).



Reshetikhin-Turaev defect TQFT

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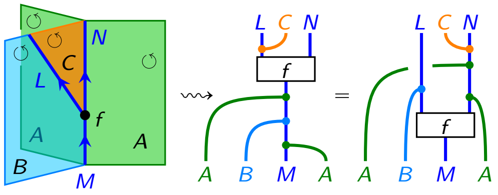
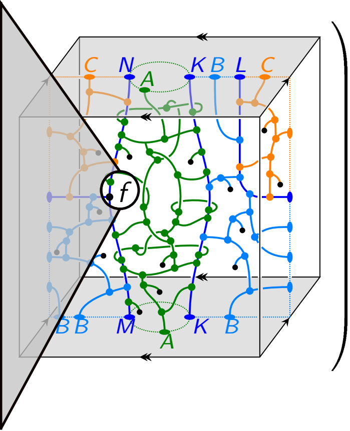
defined i.t.o. $Z^{\text{RT}}_{\mathcal{C}}$ via an internal 2d state-sum construction, e.g.

Points are labelled by multimodule morphisms, i.e. commuting with the actions of ssFA's assigned to adjacent surfaces, e.g.



(equivalently, $f : M \to L \otimes_C N$

is an $(A \otimes B)$-A-bimodule morphism).



Reshetikhin-Turaev orbifold TQFT

[Carqueville-Runkel-Schaumann'19-20]
[Carqueville-M-Runkel
-Schaumann-Scherl'21]

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$$\downarrow \text{'orbifold datum'}$$

$$Z_{\mathcal{C}}^{\text{orb}\,\mathbb{A}} \colon \widehat{\text{Bord}_3} \longrightarrow \text{Vect}$$

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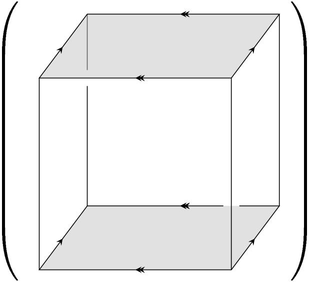
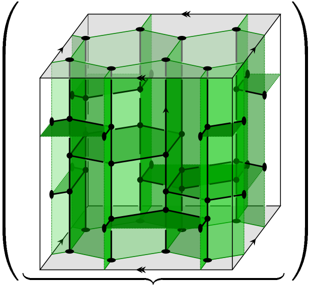
$Z_{\mathcal{C}}^{\text{orb}\,\mathbb{A}}$  $:=$ $Z_{\mathcal{C}}^{\text{def}}$ 

stratification is an admissible 2-skeleton:

- 3-strata are contractible;
- lines have 3 adjacent surfaces;
- points 4 adjacent lines
- local models + orientation constraints

(e.g. like in duals of triangulations)

Reshetikhin-Turaev orbifold TQFT

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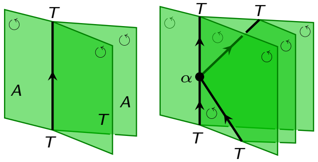
An orbifold datum $\mathbb{A} = (A, T, \alpha, \overline{\alpha}, \psi, \phi)$
carries the labels for the strata of the 2-skeleton:

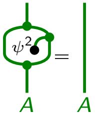


A - ssFA
${}_A T_{A\otimes A}$ - bimod.

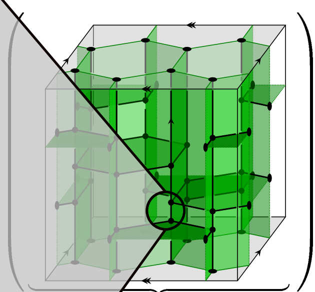
$\underbrace{\alpha : T \otimes_2 T \to T \otimes_1 T \qquad \overline{\alpha} : T \otimes_1 T \to T \otimes_2 T}_{A\text{-}(A \otimes A \otimes A)\text{-bimodule morphisms}}$

as well as some technicalities:

- $\psi \colon \mathbb{1}_{\mathcal{C}} \to A$ for the separability condition
- $\phi \in \mathbb{k}^\times$ - normalisation factor



and satisfies the conditions...



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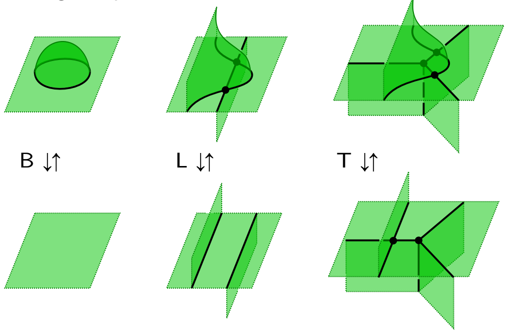
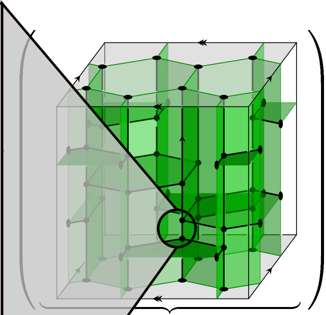
... and satisfies the conditions,
ensuring independence of the choice of 2-skeleta:



- the BLT moves must hold upon evaluating with $Z_{\mathcal{C}}^{\mathrm{def}}$;
- various (admissible) orientations ⇒ 8 conditions;
- imagine: 2-skeleton - "foam",
  BLT moves - "calculus of sliding bubbles"



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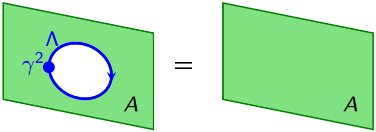
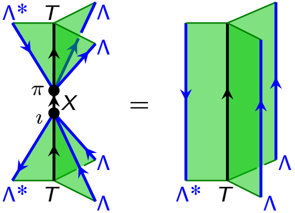
(e.g. like in duals of triangulations)

Internal Levin–Wen model: input data

1. Modular fusion category $\mathcal{C}$ with an object $X \in \mathcal{C}$
2. Orbifold datum $\mathbb{A} = (A, T, \alpha, \overline{\alpha}, \psi, \phi)$ in $\mathcal{C}$
3. A-module Λ and module endomorphism $\gamma \colon \Lambda \to \Lambda$ such that



4. split idempotent $\pi : X \rightleftarrows \Lambda^* \otimes_A T \otimes_{A \otimes A} (\Lambda \otimes \Lambda) : \imath$

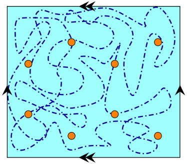


5. oriented surface Σ and an admissible skeleton $\Gamma \subseteq \Sigma$.

Internal Levin–Wen model: construction

The state space is defined to be

$$V := Z^{\mathrm{RT}}_{\mathcal{C}}\Big(\underbrace{\Sigma \text{ with anyons } X^{|v|} \in \mathcal{C} \text{ at vertices } v \in \Gamma_0}_{\text{“ } \quad \text{”}} \Big)$$



From the definition of $Z^{\mathrm{RT}}_{\mathcal{C}}$ follows:

$$V \cong \mathcal{C}\Big(\mathbb{1}, \ \bigotimes_{v \in \Gamma_0} X^{|v|} \otimes L^{\otimes g} \Big) , \qquad \underbrace{L = \bigoplus_{i \in \mathrm{Irr}_{\mathcal{C}}} U_i \otimes U_i^*}_{\text{coend}} \in \mathcal{C} \, .$$

Note: the model is not local unless $\mathcal{C} = \mathrm{Vect}_{\Bbbk}$.

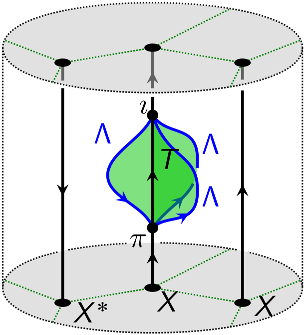
Internal Levin–Wen model: construction

The Hamiltonian is defined to be

$$H := \sum_{v\in\Gamma_0} (\mathrm{id}_V - P_v) + \sum_{v\in\Gamma_1} (\mathrm{id}_V - P_e) + \sum_{v\in\Gamma_2} (\mathrm{id}_V - P_f)$$

where

$$P_v := Z_{\mathcal{C}}^{\mathrm{def}} \Big(\quad \Big)$$



Note: since $\pi \circ \imath = \mathrm{id}$, P_v is a projector.

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where

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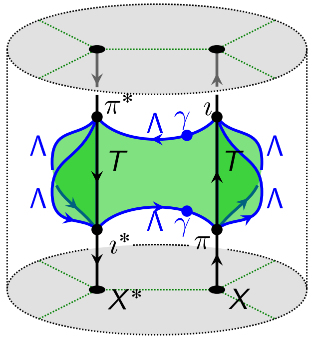

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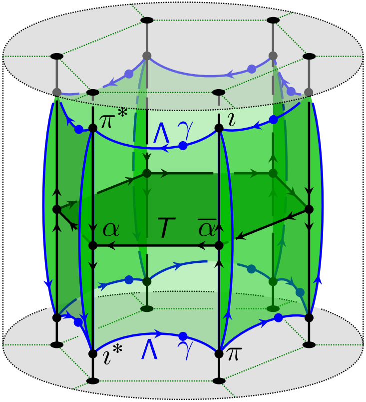
where

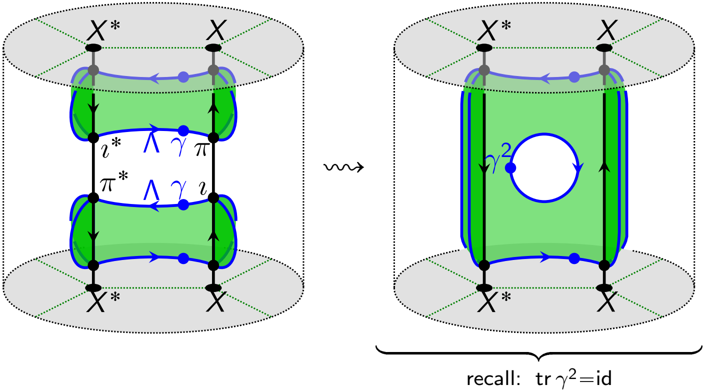
$$P_f := Z^{\mathrm{def}}_{\mathcal{C}} \left(\quad \right)$$



Properties of internal Levin–Wen model

Claim: P_v, P_e, P_f are commuting projectors.

For example $P_e P_e = P_e$ is given by



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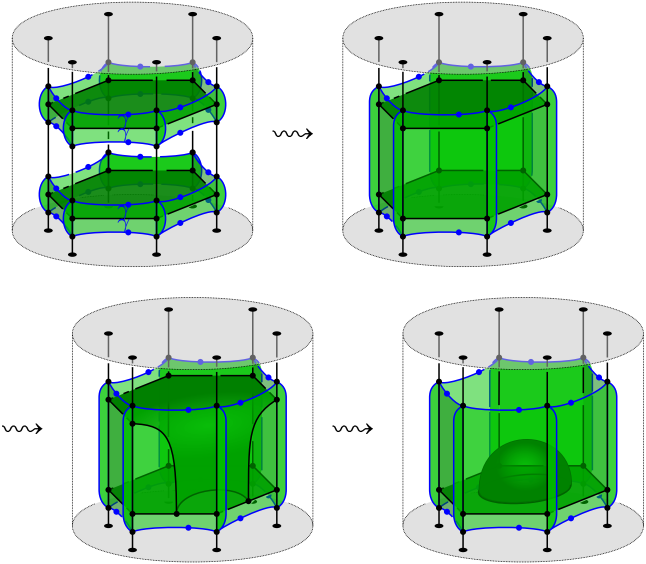
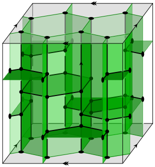


Ground state

- As in original LW have:

$$V_0 = \bigcap_{f \in \Gamma_f} \operatorname{im} P_f = \operatorname{im} \prod_{f \in \Gamma_f} P_f = \operatorname{im} Z_{\mathcal{C}}^{\mathrm{def}}\Big($$  $$\Big) = Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}(\Sigma).$$

[M-Runkel'20]

- Thm. An orbifold datum $\mathbb{A}$ in a MFC $\mathcal{C}$ yields a ribbon multifusion category $\mathcal{C}_{\mathbb{A}}$. If $\mathbb{A}$ is simple (i.e. $\mathcal{C}_{\mathbb{A}}$ is fusion $\Leftrightarrow \mathbb{1}_{\mathcal{C}_{\mathbb{A}}}$ is simple), it is a MFC.

[Carqueville-M-Runkel -Schaumann-Scherl'21]

- Thm. For $\mathbb{A}$ - simple, have an isomorphism of TQFTs $Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}} \cong Z_{\mathcal{C}_{\mathbb{A}}}^{\mathrm{RT}}$.

- $\mathcal{C}$ and $\mathcal{D}$ are Witt equivalent
$\Leftrightarrow \exists$ spherical fusion category $\mathcal{S}$, s.t. $\mathcal{C} \boxtimes \mathcal{D}^{\mathrm{rev}} \cong \mathcal{Z}(\mathcal{S})$
$\Leftrightarrow \exists$ orb. datum $\mathbb{A}$ in $\mathcal{C}$ s. t. $\mathcal{D} \simeq \mathcal{C}_{\mathbb{A}}$. [M'22]

Examples of orbifold data ($\Rightarrow$ internal LW models)

- Drinfeld doubles:
  A finite dimensional semisimple Hopf algebra K yields a simple orbifold datum in $\mathrm{Vect}_{\Bbbk}$:

$$\mathbb{A}_K = \Big(A = K_{Fr}, \quad T = K^{\otimes 2}, \quad \dots \Big).$$

K_{Fr} - ssFA w/ pairing
$h \mapsto \dim H \int h$

  Have: $(\mathrm{Vect}_{\Bbbk})_{\mathbb{A}_K} \simeq D(H) - \mathrm{Rep}$ and
  internal LW for $\mathbb{A}_K \sim$ Kitaev model.

[Carqueville-Runkel-Schaumann'19-20]
[M-Runkel'20]

- Drinfeld centres:
  A spherical fusion category $\mathcal{S}$ yields a simple orbifold datum in $\mathrm{Vect}_{\Bbbk}$:

$$\mathbb{A}_{\mathcal{S}} = \Big(A = \Bbbk^{\oplus |\mathrm{Irr}_{\mathcal{S}}|}, \; T = \bigoplus_{i,j,k \in \mathrm{Irr}_{\mathcal{S}}} \mathcal{S}(k, i \otimes j), \; \alpha, \overline{\alpha} \leftarrow F\text{-symbols of } \mathcal{S}, \; \dots \Big).$$

  Have: $(\mathrm{Vect}_{\Bbbk})_{\mathbb{A}_{\mathcal{S}}} \simeq \mathcal{Z}(\mathcal{S})$ and
  internal LW for $\mathbb{A}_{\mathcal{S}} \sim$ original LW model.

Further examples of orbifold data

- Condensations: [Carqueville-Runkel-Schaumann'19-20] [M-Runkel'20]

  $\mathcal{C}$ - modular fusion category, $B \in \mathcal{C}$ - condensable algebra (i.e. commutative haploid ssFA)
  yields a simple orbifold datum in $\mathcal{C}$:

  $$\mathbb{B} = \Big(B\,, {}_B B_{B\otimes B}\,, \alpha = \overline{\alpha} = \Big.$$  $$, \psi =$$  $$, \phi = 1 \Big)$$

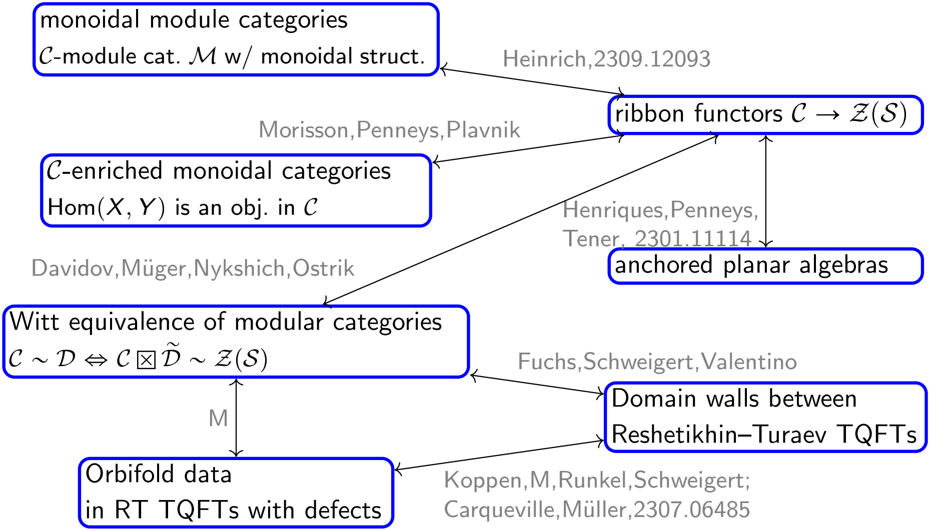
  Have: $\mathcal{C}_{\mathbb{B}} \simeq \mathcal{C}_B^{\text{loc}}$ - the category of local (dyslectic) modules

- Un-condensations: [M'22]

  Thm. There is a simple orbifold datum $\mathbb{A}$ in $\mathcal{C}_B^{\text{loc}}$ s.t. $(\mathcal{C}_B^{\text{loc}})_{\mathbb{A}} \simeq \mathcal{C}$.

  Explicit example: [M-Runkel'20]
  $\mathcal{C} = \mathcal{C}(sl_2, 10)$, $B = \underline{0} \oplus \underline{6}$ - the 'E_6 algebra'
  $\Rightarrow \mathcal{C}_B^{\text{loc}} \simeq \mathcal{I}$ - (an) Ising category.

Orbifold data vs related notions



Final remarks

- Possible future work:
  1. category $\mathcal{C}_{\mathbb{A}} \rightsquigarrow$ excitations of internal Levin–Wen;
  2. string-net description of the ground state space
     cf. [Huston–Kawagoe–Penneys–Poudel–Sanford] 2305.14068
  3. similar models in other TQFTs:
     - 2d Landau–Ginzburg [Carqueville–Murfett–Montiel-Montoya]
     - 3d Turaev–Viro [Meusburger'22]
       [Carqueville–Müller] 2307.06485
       [Lootens–Fuchs–Haegeman–Schweigert–Verstraete'20]
       Applications to tensor networks
     - 3d Rozansky–Witten [Brunner–Carqueville–Fragkos–Roggenkamp] 2307.06284
     - 4d Douglas–Reutter
     - ...
- Related notion: condensation monads [Gaiotto–Johnson-Freyd'19]

Thank you!

MFCs from orbifold data

[M-Runkel'20]

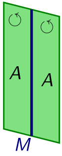
Question: Is the TQFT $Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}$ again of Reshetikhin-Turaev type?
If so, one must include embedded ribbon graphs into $Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}$

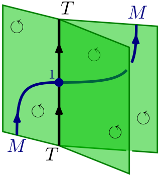
Definition

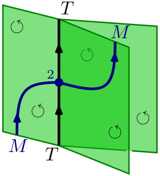
For an orbifold datum $\mathbb{A} = (A, T, \alpha, \overline{\alpha}, \psi, \phi)$, define the category $\mathcal{C}_{\mathbb{A}}$ with

- objects: tuples

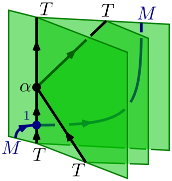
$$\left(M \in {}_A\mathcal{C}_A, \quad \tau_1 \colon M \otimes_0 T \xrightarrow{\sim} T \otimes_1 M, \quad \tau_2 \colon M \otimes_0 T \xrightarrow{\sim} T \otimes_2 M \right)$$







such that



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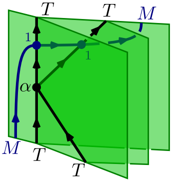
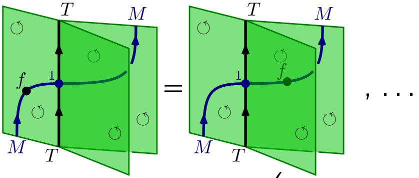


, . . . (7 conditions in total
⇒ strands are independent on
the embedding into the foam)

MFCs from orbifold data

[M-Runkel'20]

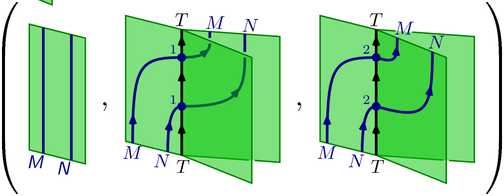
- morphisms: $f\colon M \to N$ is an A-A-bimodule morphism such that



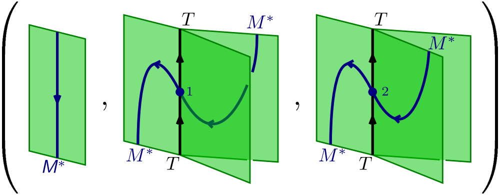
(similarly for τ_2
$\Rightarrow$ points are independent on
the embedding into the foam)

The category $\mathcal{C}_{\mathbb{A}}$ is:

- monoidal: $M \otimes N :=$



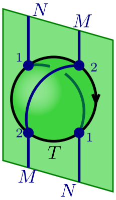
- spherical: $M^* :=$



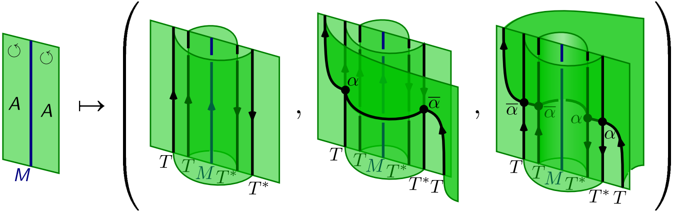
clearly pivotal like ${}_A\mathcal{C}_A$,
sphericality is shown
separately

MFCs from orbifold data

[M-Runkel'20]

- braided: $c_{M,N} := \phi^2 \cdot$ 

- finitely semisimple:
  the pipe functor $P\colon {}_A\mathcal{C}_A \to \mathcal{C}_{\mathbb{A}}$



  is the (left and right) adjoint to the forgetful functor $U\colon \mathcal{C}_{\mathbb{A}} \to {}_A\mathcal{C}_A$
  and the (bi)adjunction is separable (then ${}_A\mathcal{C}_A$ fin. s.si. $\Rightarrow \mathcal{C}_{\mathbb{A}}$ fin. s.si)

Theorem *If $\mathbb{A}$ is simple (i.e. $\mathbb{1}_{\mathcal{C}_{\mathbb{A}}} := A$ is simple) then $\mathcal{C}_{\mathbb{A}}$ is a MFC.*

Reshetikhin-Turaev orbifold graph TQFT

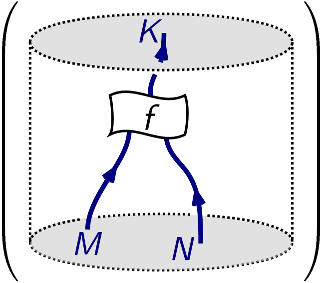
[Carqueville-M-Runkel -Schaumann-Scherl'21]

... is the symmetric monoidal functor

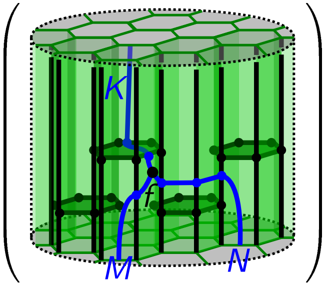
$$Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}} \colon \widehat{\mathrm{Bord}}_3(\mathcal{C}_{\mathbb{A}}) \longrightarrow \mathrm{Vect}$$

defined i.t.o. $Z_{\mathcal{C}}^{\mathrm{def}}$ via an internal 3d state-sum construction in which the ribbon graphs are embedded into the foam e.g.

$Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}$



$:=$ $Z_{\mathcal{C}}^{\mathrm{def}}$



stratification is a ribbon diagram:

- admissible 2-skeleton+ribbon graph;
- intersection points labelled by τ's;

Theorem *If $\mathbb{A}$ is simple then the graph TQFTs $Z_{\mathcal{C}_{\mathbb{A}}}^{\mathrm{RT}}$ and $Z_{\mathcal{C}}^{\mathrm{orb}\,\mathbb{A}}$ are isomorphic.*