Higher Berry Curvatures for Locally Parametrized States

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Topo23 Winter Workshop
(Work in preperation)

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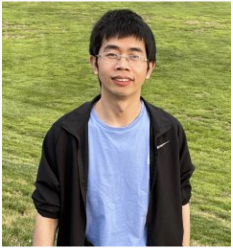
People



References

[1] Adam Artymowicz et al. **“Quantization of the higher Berry curvature and the higher Thouless pump”.** (2023).

[2] Anton Kapustin et al. **“Higher-dimensional generalizations of Berry curvature”.** (2020).

[3] Anton Kapustin et al. **“Local Noether theorem for quantum lattice systems and topological invariants of gapped states”.** (2022).

[4] A. Kitaev. **“Differential forms on the space of statistical mechanics models”.** 2019.

[5] Shuhei Ohyama et al. **“Higher structures in matrix product states”.** (2023).

[6] Marvin Qi et al. **“Charting the space of ground states with tensor networks”.** (2023).

[7] Ken Shiozaki et al. **“Higher Berry curvature from matrix product states”.** (2023).

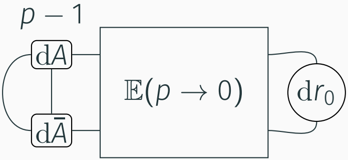
[8] Xueda Wen et al. **“Flow of (higher) Berry curvature and bulk-boundary correspondence in parametrized quantum systems”.** (2022).

Results

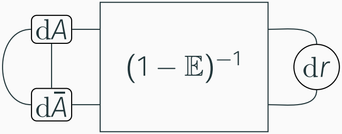
Construct from a *local parameter space* a topological invariant of family of states X in $d+1$, that is integral of $d+2$ differential form $\Omega^{(d+2)}$ over X - Higher Berry curvature

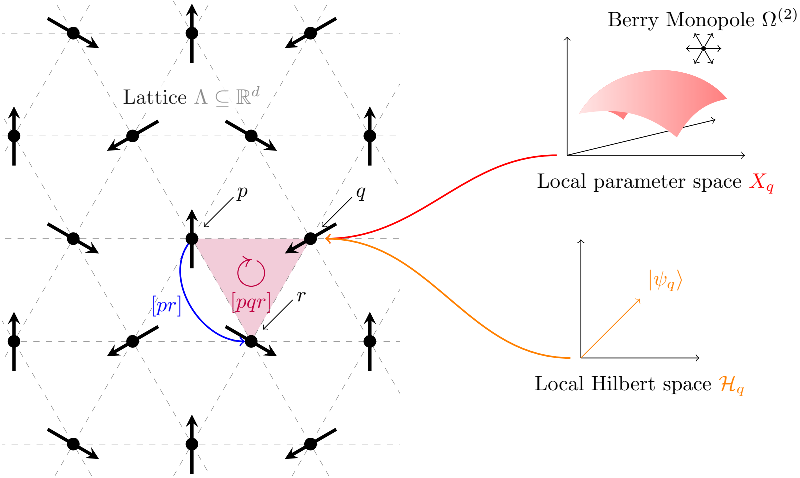
Example:

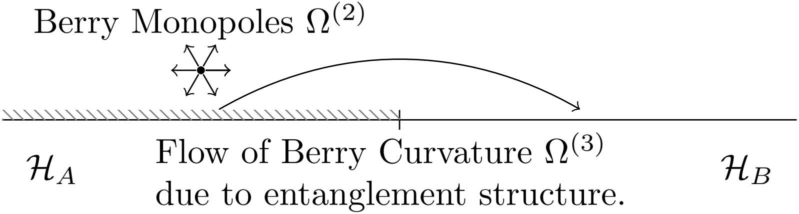
$$\Omega^{(3)} = \sum_{p \le 0} \mathrm{Im} \tag{1}$$

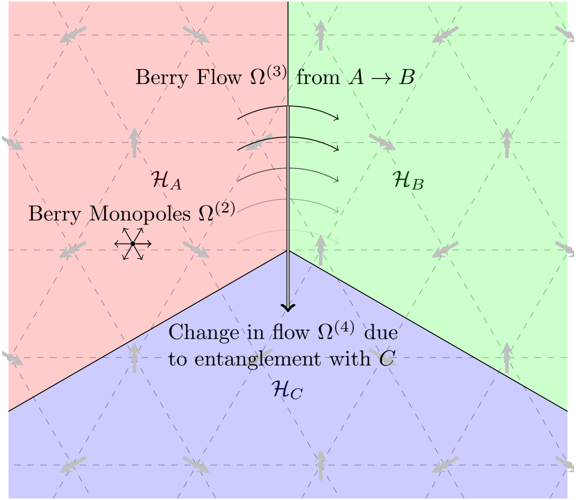


$$\overset{\text{trans.inv.}}{=} \mathrm{Im} \tag{2}$$




Berry Monopole $\Omega^{(2)}$
Lattice $\Lambda \subseteq \mathbb{R}^d$
p
q
r
$[pr]$
$[pqr]$
Local parameter space X_q
$|\psi_q\rangle$
Local Hilbert space $\mathcal{H}_q$


Berry Monopoles $\Omega^{(2)}$
$\mathcal{H}_A$
Flow of Berry Curvature $\Omega^{(3)}$
due to entanglement structure.
$\mathcal{H}_B$



Berry Flow $\Omega^{(3)}$ from $A \to B$
$\mathcal{H}_A$
$\mathcal{H}_B$
Berry Monopoles $\Omega^{(2)}$
Change in flow $\Omega^{(4)}$ due to entanglement with C
$\mathcal{H}_C$

Continuity equation

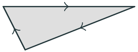
$$p_i \in \Lambda$$

$$[p_0] = \;\cdot\; p_0 \longrightarrow \text{charge(dens) } Q = Q^{(k)} = \sum_p Q_p[p]$$

$$[p_0 p_1] =$$

$$p_0 \; p_1 \longrightarrow \text{current(dens) } j = Q^{(k+1)} = \frac{1}{2} \sum_{pq} j_{pq}[pq]$$

$$[p_0 p_1 p_2] = p_2$$



$$p_0 \; p_1 \longrightarrow \text{circ. } Q^{(k+2)} = \frac{1}{3!} \sum_{pqr} Q^{(k+2)}_{pqr}[pqr]$$

$$\partial Q^{(k+1)} = \mathrm{d} Q^{(k)}$$

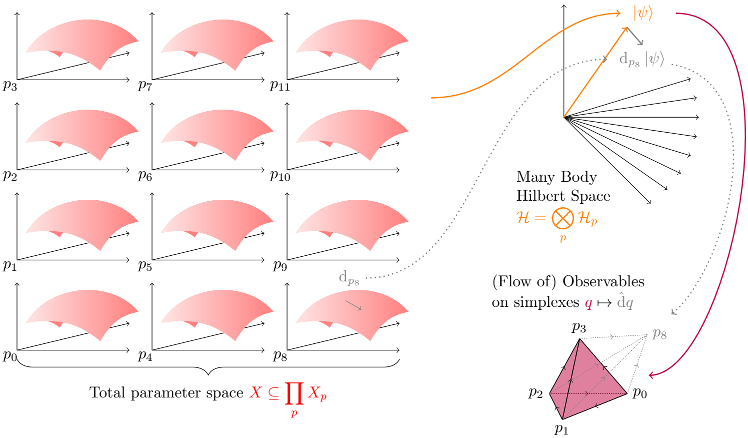
Non-trivial $\leftrightarrow$ chains with finite correlation lengths
Natural formal solutions using local structure

$$\hat{\mathrm{d}} \qquad \text{s.t.} \qquad \partial\hat{\mathrm{d}} + \hat{\mathrm{d}}\partial = \mathrm{d} \tag{3}$$

In particular

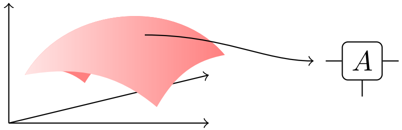
$$\hat{\mathrm{d}} : c_{p_0 \dots p_n}[p_0 \cdots p_n] \mapsto \mathrm{d}_q c_{p_0 \dots p_n}[q p_0 \cdots p_n] \tag{4}$$

Local parameter spaces $\hat{\mathrm{d}}$ acts within finite correlation length chains.

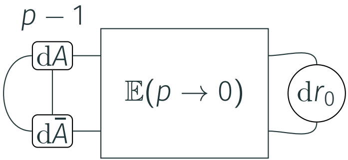

$|\psi\rangle$
$\mathrm{d}_{p_8}\,|\psi\rangle$
p_3 p_7 p_{11}
p_2 p_6 p_{10}
p_1 p_5 p_9
d_{p_8}
p_0 p_4 p_8
Many Body
Hilbert Space
$\mathcal{H} = \bigotimes_p \mathcal{H}_p$
(Flow of) Observables
on simplexes $q \mapsto \hat{\mathrm{d}}q$
p_3 p_8 p_2 p_0 p_1
Total parameter space $X \subseteq \prod_p X_p$

$$\partial Q^{(k+1)} = \mathrm{d}Q^{(k)}$$
$$Q^{(d+k)} \propto \hat{\mathrm{d}}^{d} Q^{(k)}$$

$$F^{(d+2)} \propto -\hat{\mathrm{d}}^{d+1}\,\mathrm{Im}\,\langle\psi|\mathrm{d}\psi\rangle$$
$$\Omega^{(d+2)} = \langle F^{(d+2)}, \text{“}d\text{-cut of lattice”}\rangle$$

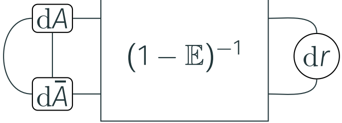


$$\Omega^{(3)} = \sum_{p\leq 0} \mathrm{Im}$$

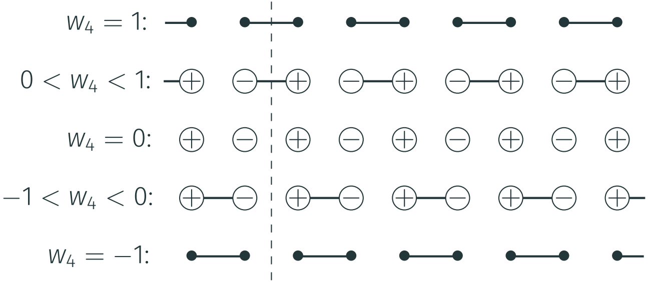


(5)

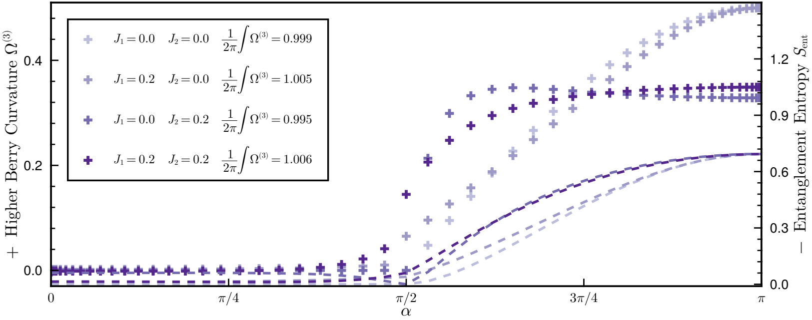
$$\overset{\text{trans.inv.}}{=} \mathrm{Im}$$



(6)



$w_4 = 1$:
$0 < w_4 < 1$:
$w_4 = 0$:
$-1 < w_4 < 0$:
$w_4 = -1$:


$J_1 = 0.0$ $J_2 = 0.0$ $\frac{1}{2\pi}\int \Omega^{(3)} = 0.999$
$J_1 = 0.2$ $J_2 = 0.0$ $\frac{1}{2\pi}\int \Omega^{(3)} = 1.005$
$J_1 = 0.0$ $J_2 = 0.2$ $\frac{1}{2\pi}\int \Omega^{(3)} = 0.995$
$J_1 = 0.2$ $J_2 = 0.2$ $\frac{1}{2\pi}\int \Omega^{(3)} = 1.006$
+ Higher Berry Curvature $\Omega^{(3)}$
— Entanglement Entropy $S_{\rm ent}$
α
0
$\pi/4$
$\pi/2$
$3\pi/4$
π
0.0
0.2
0.4
0.0
0.3
0.6
0.9
1.2

Outlook: What I'm thinking about

1. Do (injective) PEPS form local parameter spaces
2. Dynamical impact of $\Omega^{(d+2)} \neq 0$
3. (Dis)prove quantisation in $d > 1$.
4. Show that this is only interesting feature in space of phases