MPS with measurement-assisted quantum circuits: Transformations and phases of matter

Georgios Styliaris
("Yorgos")

arXiv: 2307.01696

Joint work with
D. Malz, Z.-Y. Wei, and I. Cirac

MAX PLANCK INSTITUTE
OF QUANTUM OPTICS

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Motivation: Quantum state preparation

Given the description of a quantum state, how we create it on quantum hardware?

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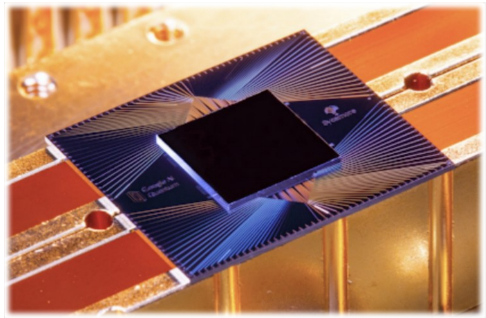
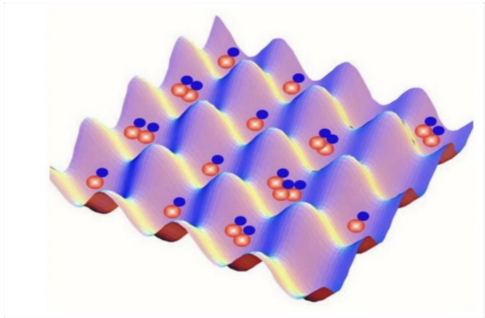


Quantum computing



Quantum simulation

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Complexity of operations is restricted by **noise** and (often) **locality**

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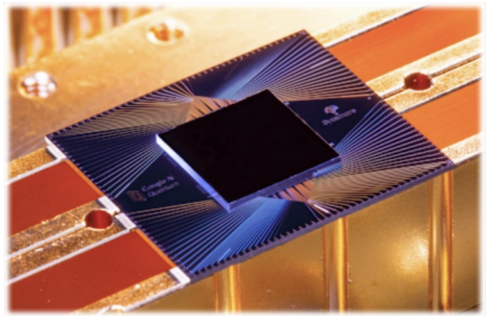

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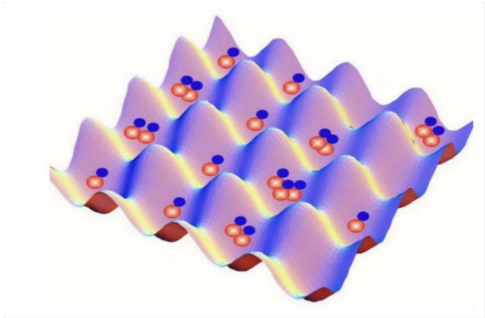

Quantum simulation

• • •

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Which states can be created with a **reasonable** amount of resources?

Topological phases, complexity, and state preparation

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Phase = Equivalence class

States in the same phase can be connected by a **shallow-depth**, **local** quantum circuit

Hastings, Wen, PRB '05
Chen, Gu, Wen PRB '11
Haah et al. FOCS18

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States in the **trivial** phase are **feasible** to prepare in a quantum simulator

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Preparation of MPS

MPS are a fundamental part of many-body systems in 1D:

- Express *ground states* of local Hamiltonians
- Topological Phases *classification*
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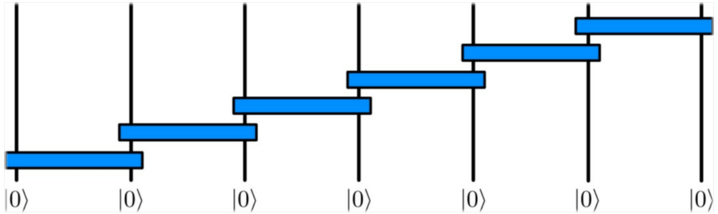
How can MPS be prepared?

Sequential preparation

- Applies to *every* MPS
- Depth T = O(N)
- Exact and explicit



Schön et al. PRA ‘07

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Schön et al. PRA '07

Adiabatic preparation

- *Injective* MPS (trivial phase)
- Depth T = O(Polylog(N/ε))
- Implicit (Hamiltonian simulation)

Ge et al. PRL '16
Bachmann et al. CMP '18

Preparation of MPS

What are the **limits** to MPS preparation?

- *What is the best possible scaling for injective MPS?*
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Is it possible to **connect phases** without a blowup in the complexity?

- *How can measurements speed-up state preparation?*

Briegel et al., PRL ‘01
Raussendorf et al, PRA ‘05
Aguado et al., PRL ‘08

Piroli et al., PRL ‘21
Tantivasadakarn et al., ‘21
Lu et al., PRXQ ‘22

Main Results

i) **Lower bound** on the complexity of preparing **injective** MPS:

- It is **impossible** to faithfully prepare any translational-invariant **injective** MPS over N sites with a local quantum circuit of depth **o(log N)**

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ii) Introduce an explicit algorithm for preparing **injective MPS** with the **optimal possible asymptotic scaling O(log N)**

- *Establishes the exact circuit complexity of injective MPS*
- *Key technical tool is MPS renormalization*

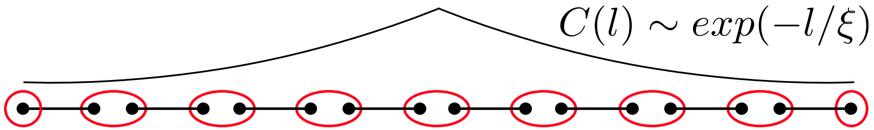
Main Results

iii) Adapt the algorithm to include **measurements**. Then **any** MPS can be prepared in:

- O(log N) depth and 1-round of measurements, or
- O(log log N) depth and O(log N) rounds of measurements

Lower bound for the preparation of injective MPS

Target state
(injective MPS)



Lower bound for the preparation of injective MPS

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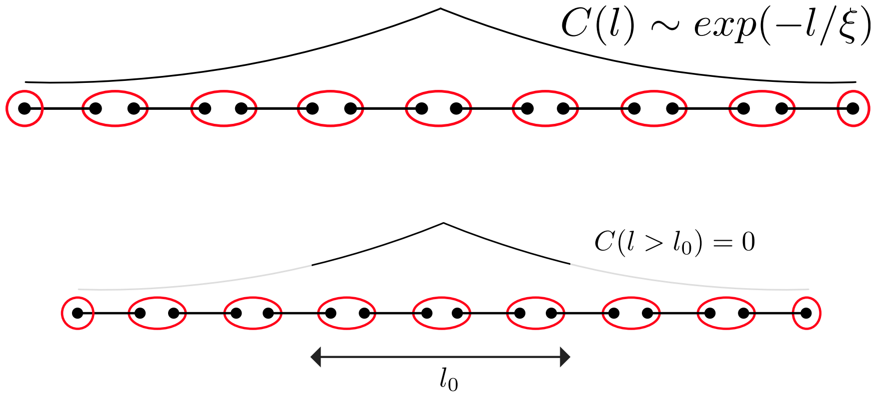
Finite depth circuit approximation
(strict light cone)



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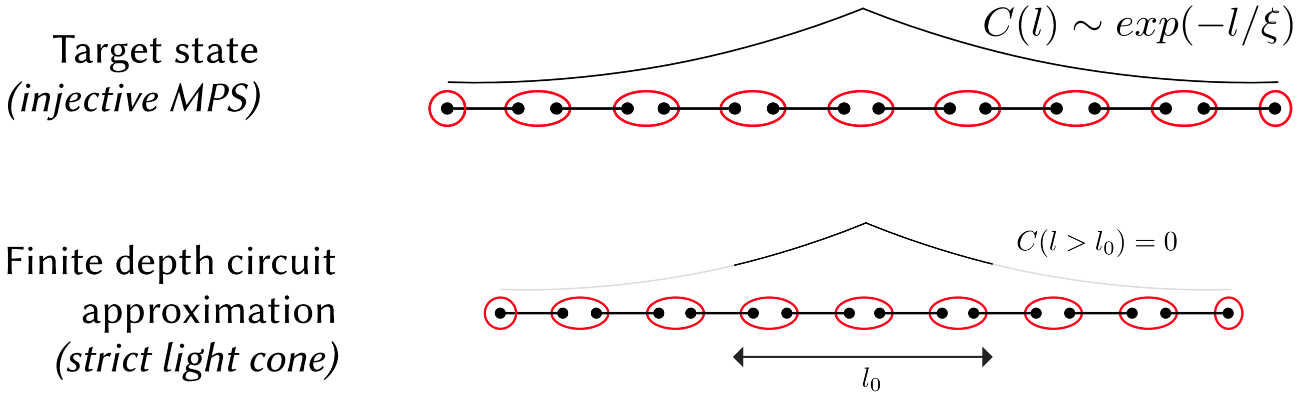
Lower bound for the preparation of injective MPS



Difference in correlations is key to prove **complexity lower bound**

Theorem. Given

- a sequence of normal TI-MPS $\{|\phi_N\rangle\}_N$ with nonzero correlation length, and
- a sequence $\{|\psi_N\rangle\}_N$ of outputs from a local quantum circuit of depth T applies to product states, then:

If T = o(log N), there exists N_0 such that for all N > N_0,
we have $1 - |\langle\phi_N|\psi_N\rangle| > 1/2$.

Preparing MPS with renormalization group (RG) transformation

Goal: Given an injective tensor A, prepare the corresponding MPS over N sites with O(log N) depth

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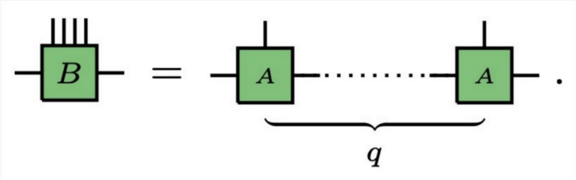

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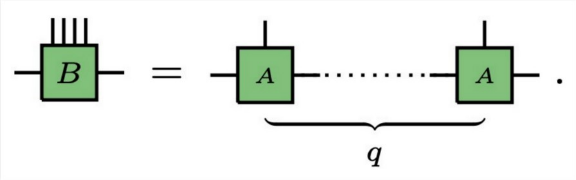

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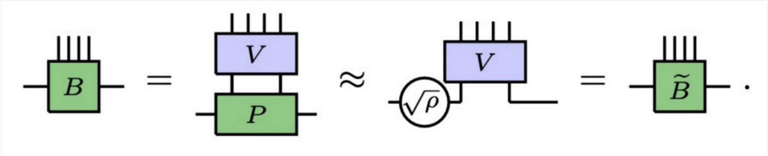

(ii) Polar decomposition and approximate with the RG fixed point

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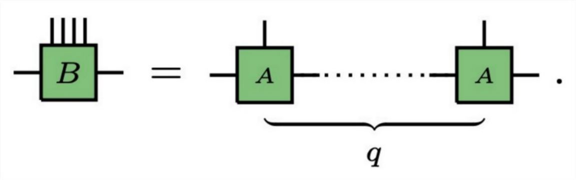

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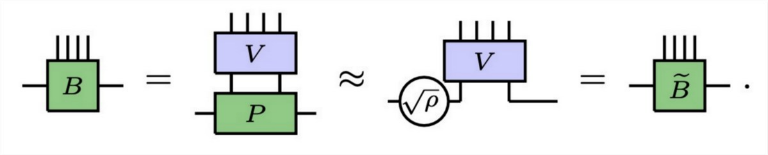

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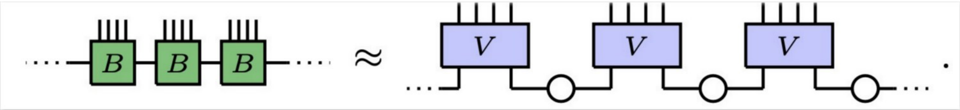

(iii) Approximate state

__**Theorem**__: Blocking **q= 2ξ log(N)** sites suffices to have a **vanishing error** in the thermodynamic limit

Piroli, **GS**, Cirac, PRL '21

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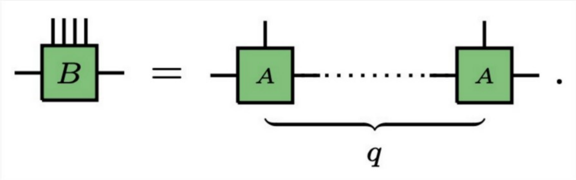

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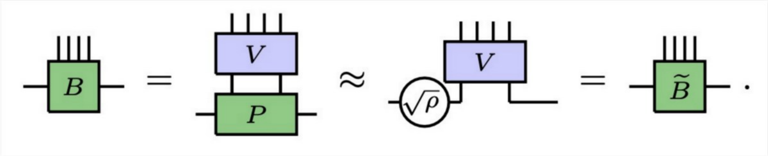

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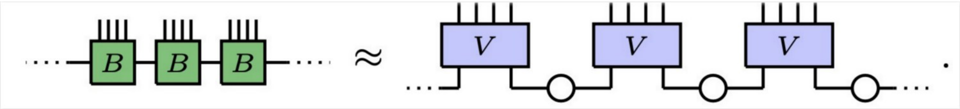

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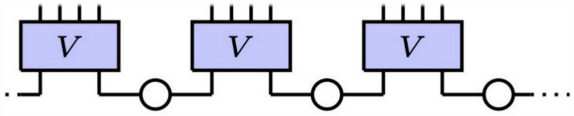
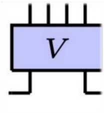
How hard is to implement the isometry V?

Piroli, **GS**, Cirac, PRL '21

Efficient implementation of the isometry

Protocol: Create entangled pairs and apply isometries 

Challenge: Implement the isometry 

over output ~log N sites with O(log N) depth

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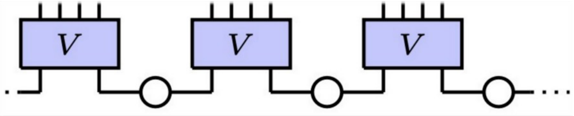
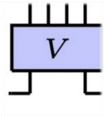


Challenge: Implement the isometry 

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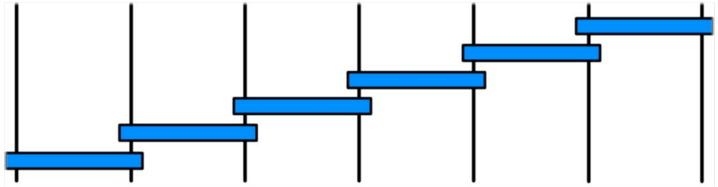
Sequential Scheme



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- Each gate has support over at most d D^2 sites

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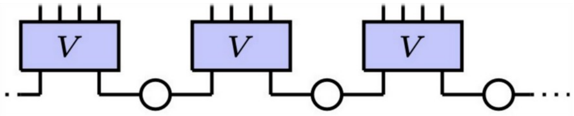
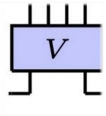


Challenge: Implement the isometry 

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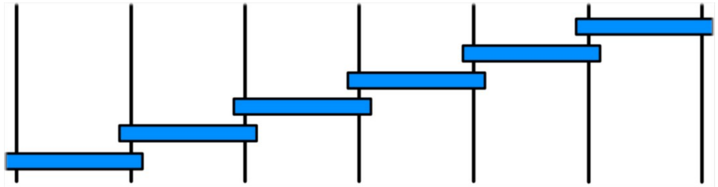

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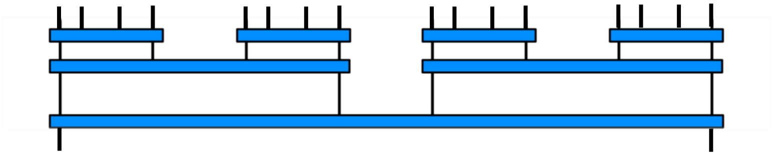

Each V has:

- O(log log N) layers (long-range gates)
- O(log N) depth for local gates

Exponential Speed-up using measurements

Injectivity implies short-range RG fixed-point: 

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For a **general** tensor,
RG fixed point has a **long-range** part:

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Sequential Scheme

Measurements are used only for the creation of the fixed point.

- Each isometry takes O(log N) depth (no measurements)
- Depth O(log N) with single round of measurements

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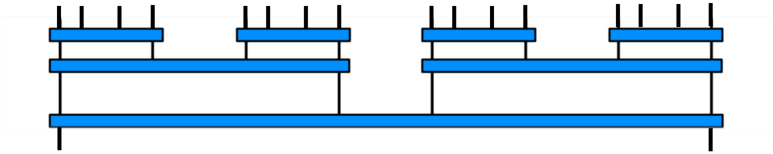

Use teleportation to implement long-range gates.

- Each layer now takes constant time.
- Depth O(log log N) with O(log N) rounds of measurements

Lu et al., PRXQ '22

Inhomogeneous MPS

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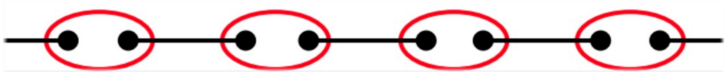
For inhomogeneous MPS, injectivity alone does not guarantee finite correlation length

"Definition" (Short-range correlated inhomogeneous MPS): A sequence of MPS $\{|\phi_N\rangle\}_N$ is short-range correlated if blocking q=O(log N) sites the state can be well-approximated, up to local isometries, by:

$$|\Omega\rangle_N = \bigotimes_{i=1}^{N/q} |\omega_j\rangle_{R_i L_{i+1}}$$



Inhomogeneous MPS

Are "generic" inhomogeneous MPS short-range correlated?

Yes!

Garnerone, Oliveira, Zanardi, PRA '10
Chen, Gu, Wen, PRB '11
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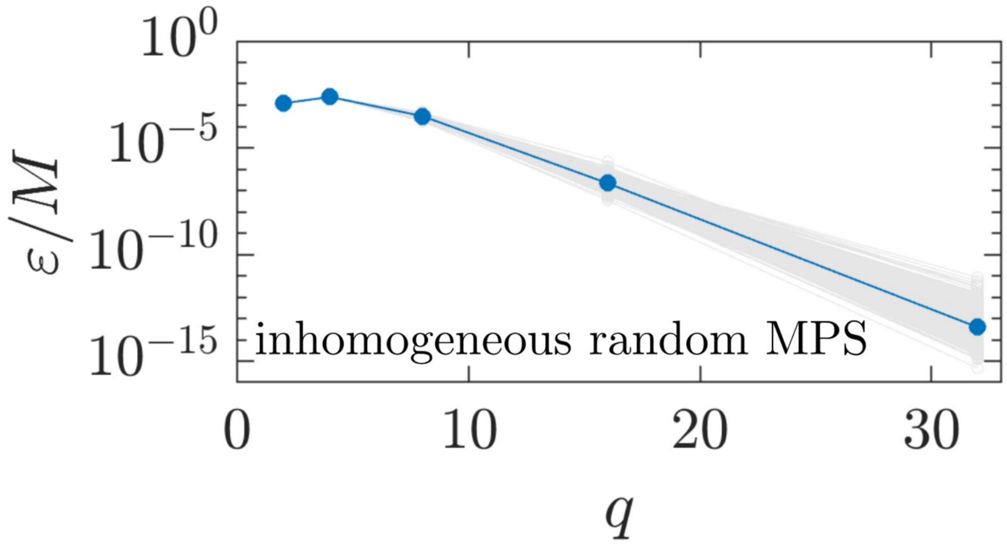

Haar random D = 2 MPS tensors

Error of the state drops exponentially in the size of block q as in translation-invariant

Summary and outlook

- Measurements change the preparation complexity of MPS (and topological states in higher spatial dimensions!)
- Lower bound $\Omega(\log N)$ for the circuit complexity of injective MPS (i.e., trivial phase of 1D gapped local Hamiltonian ground states).
- Explicit algorithm to saturate the bound $\longrightarrow$ optimal asymptotic scaling. Improves $O(\text{Polylog } N)$ depth (adiabatic preparation) to $O(\log N)$.
- Exponential speedup with measurements to depth $O(\log \log N)$ for any MPS.

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Future directions: **PEPS**?

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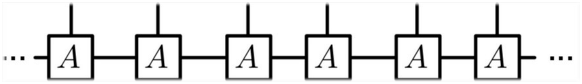
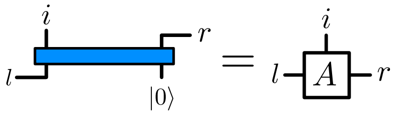
Thank you!

Sequential generation of MPS

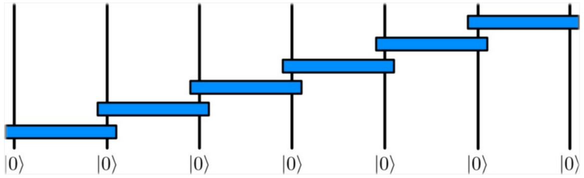
isometry

MPS in canonical form





Sequential circuit can create
long-range correlations



Linear depth is necessary for GHZ and other long-range entangled MPS

However, ground states of 1D gapped local Hamiltonians
(and injective MPS) have **exponentially-decaying correlations**

Bravyi et al., PRL '06